Bribery in Balanced Knockout Tournaments

Extended Abstract

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ABSTRACT

Balanced knockout tournaments comprise a common format for sporting competitions and pairwise decision-making. In this paper, we investigate the computational complexity of arranging the tournament's initial seeding and bribing players to guarantee one player's victory. We give a model of bribery in which the organizer can both arrange the seeding and bribe players to decrease their probability of beating other players at a cost, without exceeding a budget. We also show that it is NP-hard to determine a bribery and a seeding under which a given player wins the tournament with probability 1, even when the pre-bribery matrix is monotonic, and the post-bribery matrix is ϵ-monotonic and very close to the initial one. We also show that for almost all n player inputs generated by a well known deterministic model due to Condorcet, one can always bribe the "top" $O(\log n)$ players so that there is an efficiently constructible seeding for which *any* player wins.

KEYWORDS

Social choice theory; Other

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1 INTRODUCTION

Knockout tournaments are a popular format to determine a winner from several players over the course of several rounds until only one alternative remains. Much work in computational social choice investigates how much power a tournament organizer has over who wins by selecting the *seeding*, the permutation of the players for the first round. This is given more formally as the Tournament Fixing Problem (TFP) with input (i^*, P, δ), where $\delta \in [0, 1]$, $i^* \in [n]$ is a "favorite" player and P is an $n \times n$ matrix where the entry $P_{i,j}$ gives the probability that player i *beats* player j. TFP asks whether a self-interested organizer can select a seeding s for which i^* wins the balanced knockout tournament with probability at least δ.

In its general form, TFP has been known to be NP-complete for over a decade ([8]). Several restricted versions of TFP (such as the deterministic case) are also known to be NP-hard [2], and some recent works have tested this hardness result empirically with real-world data [10] [11]. Others have worked to formulate the complexity of certain versions using FPT algorithms [4] [1].

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In this paper we consider the TFP problem when the tournament organizer can bribe players, and P is deterministic or $\varepsilon-$*monotonic*: the players are ordered from 1 (the strongest) to n (the weakest), and for every i and j, $P_{i,j} \geq P_{i,j-1} - \varepsilon$; 0-monotonic is just called monotonic. Notions of bribery in knockout tournaments have been studied before, e.g. by [5], [7], [9], and [12]. Our work extends what is known in novel ways.

1.1 Our Results

We first formulate a new bribery version of TFP called BTFP, extending a notion from [7]. Generally, to show that BTFP is NP-hard, we reduce TFP to it [15]. This does not work when P is monotonic because TFP is not known to be NP-hard in that case. We show that BTFP is NP-complete even when P is monotonic using a new reduction where each entry of the post-bribery matrix P' differs from the corresponding entry in P by no more than an arbitrary small constant $\varepsilon > 0$, which allows the manipulator performing the bribes to eschew getting caught.

Next, we consider the case where for every pair of players i and j, $P_{ij} \in \{0, 1\}$. Bribery problems for deterministic P have been proposed in the past. [12] and [5] incorporated bribery into the simpler problem of modifying a winning seeding, and [7] bounded the number of players that can be bribed from above to show that a special case of BTFP is NP-hard. We want to bribe better so that (1) the post-bribery matrix does not depend heavily on the initial seeding or favorite player, and therefore does not look suspicious; (2) we do not have to re-bribe if we pick a new favorite player; and (3) our input matrix P is more realistic, with stronger players generally beating weaker ones. We consider matrices P generated by a natural model proposed by a Condorcet. We show that for almost all such matrices, the top $O(\log n)$ players can always be bribed to throw certain matches so that any player can win.

2 BRIBERY MODEL

We define a model for bribing players competing in a balanced knockout tournament KT. It is assumed that for any pairwise match, the probability of one player winning against the other is known, regardless of whether or not a bribe was made. We are given the following as inputs: a set of players $S = [n]$; a probabilistic pairwise comparison matrix P; a matrix $A \in \mathbb{Q}^{n \times n}_{[0,1]}$ where $A_{i,j}$ denotes the probability that row player i will beat column player j in a match that j has been bribed to throw, and for all (i, j), $1 \geq A_{i,j} \geq P_{i,j}$; a cost matrix $C \in \mathbb{Z}^{(n \times n)}$ where $C_{i,j}$ denotes the cost of bribing column player j to throw the match to row player i; a budget $B \in \mathbb{Z}^+$, the total amount that can be spent on bribing players; a favorite player $v^* \in S$; and a threshold probability $\delta \in \mathbb{Q}_{[0,1]}$.

	v^*	v_j	e_j	$f_j^{r'}$	$h_{e_j}^{t'}$	$h_{f_j^{r'}}^{t'}$	$h_*^{t'}$
v^*	$-$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$
v_i	ϵ	$1-\epsilon$ if $i < j$, ϵ o.w.	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$
e_i	ϵ	ϵ	ϕ	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$
f_i^r	ϵ	ϵ	ϵ	ϕ	ϕ	ϕ	$1-\epsilon$
$h_{e_i}^t$	ϵ	ϵ	ϵ	ϕ	ϕ	ϕ	$1-\epsilon$
$h_{f_i^r}^t$	ϵ	ϵ	ϵ	ϕ	ϕ	ϕ	$1-\epsilon$
h_*^t	ϵ	ϵ	ϵ	ϵ	ϵ	ϵ	ϕ

	v^*	v_j	e_j	$f_j^{r'}$	$h_{e_j}^{t'}$	$h_{f_j^{r'}}^{t'}$	$h_*^{t'}$
v^*	$-$	1	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	1
v_i	0	1 if $i < j$, 0 o.w.	1 if v_i covers e_j, $1-\epsilon$ o.w.	1	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$
e_i	ϵ	0 if v_j covers e_i, ϵ o.w.	ϕ	1	1 if $i = j$, $1-\epsilon$ o.w.	$1-\epsilon$	$1-\epsilon$
f_i^r	ϵ	0	0	ϕ	ϕ	1 if $f_i^r = f_j^{r'}$, ϕ o.w.	$1-\epsilon$
$h_{e_i}^t$	ϵ	ϵ	0 if $i = j$, ϵ o.w.	ϕ	ϕ	ϕ	$1-\epsilon$
$h_{f_i^r}^t$	ϵ	ϵ	ϵ	0 if $f_i^r = f_j^{r'}$, ϕ o.w.	ϕ	ϕ	$1-\epsilon$
h_*^t	0	ϵ	ϵ	ϵ	ϵ	ϵ	ϕ

Table 1: (a) On the left is matrix P. (b) On the right is the resulting matrix P' post-bribery. The bribes are indicated by the colored cells

With this model, we define the *tournament fixing problem with bribery* (BTFP): "Given S, P, C, A, B, v^*, and δ, is there a seeding for a balanced single-elimination tournament KT so that v^* will win with probability $\geq \delta$ if we bribe players according to C without exceeding B?" We consider two cases of this problem: one with monotonic P, and one with deterministic P and A, and where $\forall(i,j), A_{i,j} = 1$ and $C_{i,j} = 1$.

3 HARDNESS PROOF

We state and prove the complexity of BTFP. It is not hard to show NP-completeness when P is ϵ-monotonic. Set $A = P$, $B = 0$, and $C = \mathbf{0}$. This turns BTFP into an instance of TFP, which has been proven to be NP-complete when P is ϵ-monotonic ([15]). However, since the hardness of TFP given monotonic P is still open, we give a new reduction to show that BTFP is NP-complete in the case where P is monotonic and the matrix P' after bribery is ϵ-monotonic.

Theorem 3.1. *BTFP is NP-complete even if P is monotonic and the matrix P' after bribery is ϵ-monotonic.*

Proof. It is easy to see that BTFP $\in$ NP. To prove that BTFP is NP-hard, we reduce from VERTEX-COVER: "Given a graph $G = (V, E)$ and $k \in \mathbb{Z}$, is there a subset $V' \in V$ such that $|V'| \leq k$ and V' covers E?" We show that one can choose an initial seeding for a tournament KT with players that must be bribed so that a special player v^* will always win if and only if G has a vertex cover of size at least k. Our proof is based on past work proving that TFP is NP-complete when P is ϵ-monotonic by reducing from VERTEX-COVER ([15]).

Given an instance (G, k) of VERTEX-COVER, we construct an instance of BTFP with the following players: (1) vertex players $\{v_i \in V\}$, and an extra player $v_0 \notin V$, which does not cover any edges; (2) a favorite player $v^* \notin V$; (3) edge players $\{e_i \in E\}$; (4) a filler player $\{f_{v_i}^r\}$ for each vertex player $v_i \in V \cup \{v^*, v_0\}$ and round $0 < r \leq \lceil \log(|V| - k) \rceil$; (5) a filler player $\{f_{e_i}^r\}$ for each $e_i \in E$ and round r such that $\lceil \log(|V| - k) \rceil < r \leq \lceil \log(|V| - k) \rceil + \lceil \log(|E|) \rceil$; (6) $2^{\lceil \log(|V| - k) \rceil} - 1$ holder players $h_{e_i}^t$ for each edge player e_i; (7) $2^r - 1$ holder players $h_{f_i^r}^t$ for each filler player f_i^r placed at round r; and (8) $2^N - 1$ holder players h_*^t for v^* where $N = \lceil \log(|V| - k) \rceil + \lceil \log(|E|) \rceil + \lceil \log(k+1) \rceil + 1$.

Let P be as in Table 1a, and let A be the matrix that is the same as P' in Table 1b above the diagonal, and P below the diagonal. Set B to be $|V|^2$. For each pair (i, j) if $i < j$ and $P_{i,j} \neq P'_{i,j}$, set the cost $C_{i,j} = 1$; otherwise set $C_{i,j} = B + 1$. If all bribes indicated in the colored cells are made, P' is an ϵ-monotonic matrix of winning probabilities and matches the matrix obtained in [15]. In the full version, we prove that v^* can win with probability 1 iff these bribes are made and there exists a vertex cover of size k in G. □

4 BRIBING A FEW TOP PLAYERS SUFFICES

We consider a simple deterministic version of BTFP: Given a probability matrix T ($T_{i,j} = 1 - T_{j,i}, T_{i,j} \in \{0, 1\}, \forall i, j \in [n]$) and a favorite player v^*, we want to bribe a small number of players to each lose a single match, at cost 1 each, and to find a seeding for which v^* wins. T is generated by the Condorcet Random model (CR Model). Given a probability $p \leq 1/2$, an $n \times n$ matrix T is generated for the players $\{1, \ldots, n\}$ so that for all $i < j$, independently, $T_{i,j} = 1$ with probability $1 - p$ and $T_{i,j} = 0$ with probability p ([3]; [14]; [13]; [6]). Intuitively, this means that a stronger player beats a weaker player except with small probability p. Our main theorem is

Theorem 4.1. *Let n be a power of 2, $c > 1$, let $p \leq 1/2$ be arbitrary, and let $R \approx 5.92$. Then, for at least a $1 - 1/n^{c-1}$ fraction of all $n \times n$ T generated by the CR model with probability p, and any player v^*, if one bribes players $1, \ldots, Rc \log n$ to lose to v^*, there is an efficiently constructible seeding for which v^* wins.*

We show that for $v^* = n$, if we bribe players $1, \ldots, 6c \log n$ to lose to v^*, then v^* fulfills the conditions of the following theorem with probability at least $1 - 1/n^{c-1}$ [6].

Theorem 4.2. *Consider a tournament graph T on n players V where $\mathcal{K} \in V$ is a king. Let $A = N_{out}(\mathcal{K})$ and $B = V \setminus (A \cup \{\mathcal{K}\}) = N_{in}(\mathcal{K})$. Suppose that B is a disjoint union of three (possibly empty) sets H, I, J such that (1) $|H| < |A|$, (2) $in_A(i) \geq \log n$ for all $i \in I$, and (3) $out(j) \leq |A|$ for all $j \in J$. Then there exists an efficiently computable winning seeding for $\mathcal{K}$.*

The full proof shows that for any player v^*, if we bribe players $[1, , 6c \log n]$ to lose to v^*, the conditions of Theorem 4.2 are met with probability at least $1 - 1/n^{c-1}$. In particular, we prove the second condition of Theorem 4.2 using a union bound and a Chernoff-based concentration bound for the sum of Bernoulli random variables.

5 CONCLUSIONS AND FUTURE WORK

We proved that fixing a knockout tournament with monotonic P using bribery is NP-complete. The problem of fixing a tournament with monotonic P without bribery is still open. We also proved that a knockout tournament described by the CR model almost always has an optimally small set of players that can be bribed. Possible future work includes using the size of this set to parameterize BTFP.

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