Probabilistic Resource-bounded Alternating-time Temporal Logic

Extended Abstract

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ABSTRACT

This paper extends resource-bounded ATL with probabilistic reasoning and provides the syntax and semantics of the resulting logic, probabilistic resource-bounded ATL.

KEYWORDS

Logics for resource-bounded agents, Probabilistic reasoning.

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1 INTRODUCTION

In the literature, several variants of ATL and Coalition Logic have been proposed (see e.g., [1, 8, 13]). These logics allow us to express many interesting properties of coalitions and strategies, such as "*a coalition of agents $A(\subseteq N)$ has a strategy to reach a state satisfying φ no matter what the other agents $(N \setminus A)$ in the system do*", where φ characterises, e.g., lifting up a heavyweight (by a group of robots A) or a solution to a problem. However, there is no natural way of expressing resource requirements in these logics. The resource-bounded alternating-time temporal logic (RB-ATL) [3] was developed for reasoning about coalitional ability under resource bounds. RB-ATL allows us to express various resource-bounded properties, including, for example, *"coalition A has a strategy to reach a state satisfying φ under the resource bound b, but they cannot enforce φ under a tighter resource bound b'"*. In fact, the logic mentioned above can be used to state various qualitative properties of real-world systems. However, it is also equally or even more important to analyse quantitative properties of systems, such as reliability and uncertainty, which cannot be trivially expressed in logics mentioned above. A large number of multi-agent application domains, such as Internet of Things (IoT) and Cyber-Physical Systems (CPS) in general and disaster rescue and military operations in particular, require not only the reasoning about the team behavior of agents but also that the agents and/or the environment may have random or unreliable behaviours [6, 14]. Their applications encompass many safety-critical domains, and many such applications run in resource-constrained devices and environments [2, 12]. Therefore, these systems often need rigorous analysis and verification to ensure the correctness of their designs [10]. In such domains, the behaviour of an agent has to be described in terms of a distribution of probability over a set of possibilities. There has recently been increasing interest in developing logics incorporating probabilistic reasoning [5, 7, 9], which are essentially extensions of CTL or ATL. In rPATL [5], we can express that a coalition of agents has a strategy which can ensure that either the probability of an event's occurrence or an expected reward measure meets some threshold. However, probabilistic resource-bounded properties such as, for example, "*can coalition A has a strategy so that the probability to reach a state satisfying φ under the resource bound b is at least 0.95*" can neither be expressed in rPATL nor in any other probabilistic temporal logics mentioned above. In this paper, we propose a logic Probabilistic Resource-Bounded ATL (pRB-ATL) which allows us to express such properties.

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2 SYNTAX AND SEMANTICS OF PRB-ATL

In this section, we provide the syntax and semantics of the proposed logic pRB-ATL. Let $N = \{1, 2, \dots, n\}$ be a set of $n(\geq 1)$ agents forming a multi-agent system. Let R be a set of resources (e.g., money, energy, etc.). We assume that a cost of an action, for each of the resources, is a natural number. The set of resource bounds $\mathbb{B}$ over R is defined as $\mathbb{B} = (\mathbb{N} \cup \{\infty\})^r$, where $r = |R|$. Let Q be a finite set and $\delta : Q \longrightarrow [0, 1]$ be *probability distribution* function over Q such that $\sum_{q \in Q} \delta(q) = 1$. We denote by $\mathcal{D}(Q)$ the set of all such distributions over Q. For a given $\delta \in \mathcal{D}(Q)$, $supp(\delta) = \{q \in Q \mid \delta(q) > 0\}$ is called the *support* of δ. The interested reader is referred to [4] for a complete description relating to probability distributions and measures.

2.1 Syntax of pRB-ATL

Let Π be a finite set of atomic propositions, N be the set of agents, A be a non-empty subset of N, and $b \in \mathbb{B}$. The syntax of *pRB−ATL* is defined as follows:

$$\varphi := \top \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle\langle A^b \rangle\rangle \mathsf{P}_{\bowtie v}[\psi]$$

$$\psi := \bigcirc\varphi \mid \varphi\, \mathcal{U}^k\, \varphi \mid \neg\psi$$

where $p \in \Pi$, $\bowtie \in \{<, \leq, =, \geq, >\}$, $v \in \mathbb{Q} \cap [0, 1]$, $k \in \mathbb{N} \cup \{\infty\}$.

The two temporal operators have standard meaning $\bigcirc$ for "next" and $\mathcal{U}^{\leq k}$ for "bounded until" if $k \leq \infty$ or "until" otherwise. When $k = \infty$, we write $\mathcal{U}$ instead of $\mathcal{U}^\infty$. Here, $\langle\langle A^b \rangle\rangle \mathsf{P}_{\bowtie v}[\bigcirc\varphi]$ means that a coalition A has a strategy to make sure that the next state satisfies φ under resource bound b with a probability in relation $\bowtie$ with constant v. The formula $\langle\langle A^b \rangle\rangle \mathsf{P}_{\bowtie v}[\varphi_1\, \mathcal{U}\, \varphi_2]$ means that A has a strategy to enforce φ_2 while maintaining the truth of φ_1, and the cost of this strategy is at most b with a probability in relation $\bowtie$ with constant v. Other classical abbreviations for $\bot$, $\vee$, $\rightarrow$ and $\leftrightarrow$ and temporal operations are defined as usual.

2.2 Semantics of pRB-ATL

We extend the definition of resource-bounded concurrent game structures [3] with probabilistic behaviours of agents.

Definition 2.1. A probabilistic resource Concurrent Game Structure (pRCGS) is a tuple $S = (n, r, Q, \Pi, \pi, d, c, \delta)$ where:

- $n \geq 1$ is the number of agents;
- $r \geq 0$ is the number of resources;
- Q is a non-empty finite set of states;
- Π is a finite set of propositional variables;
- $\pi : \Pi \to \wp(Q)$ is a function which assigns to each variable in Π a subset of Q;
- $d : Q \times N \to \mathbb{N}^+$ is a function which indicates the number of actions available at a state for each agent where $\mathbb{N}^+ = \mathbb{N} \backslash \{0\}$;
- $c : Q \times N \times \mathbb{N}^+ \to \mathbb{B}$ is a partial function which indicates a resource cost for each action by an agent at a state; furthermore $c(q, a, 1) = \vec{0}$ for any $q \in Q$ and $a \in N$;
- $\delta : Q \times (N \to \mathbb{N}^+) \to \mathcal{D}(Q)$ is a partial probabilistic transition function (also known as probability transition matrix).

Given a pRCGS $S = (n, r, Q, \Pi, \pi, d, c, \delta)$, we identify available actions at a state $q \in Q$ of an agent $a \in N$ by numbers $1, \ldots, d(q, a)$ where $D_a(q)$ denotes the set of available actions $\{1, \ldots, d(q, a)\}$; action 1 specifies idling which is always available and cost of $\vec{0}$ by definition. Given a coalition $A \subseteq N$, a joint action m of A is a function $m : A \to \mathbb{N}^+$. Let D denote the set of all joint actions for the grand coalition N, i.e., $D = N \to \mathbb{N}^+$. Given a state q, the set of available joint actions of A at q is denoted by $D_A(q) = \{m : A \to \mathbb{N}^+ \mid \forall a \in A : m(a) \in D_a(q)\}$. The cost of an available joint action $m \in D_A(q)$ is defined as $cost(q, m) = \sum_{a \in A} c(q, a, m(a))$. When $A = N$, $D_A(q)$ is written as $D(q)$. Given a joint action $m \in D_A(q)$, the cost of m is defined as $cost(q, m) = \sum_{a \in A} c(q, a, m(a))$, i.e., the total cost of actions by agents in the coalition.

Given a pRCGS S, an infinite run(computation) is an infinite sequence $\lambda = q_0 \xrightarrow{m_0} q_1 \xrightarrow{m_1} \ldots \in (Q \times D)^\omega$ where $m_i \in D(q_i)$ and $\delta(q_i, m_i)(q_{i+1}) > 0$ for all $i \geq 0$. We denote the set of all infinite computations by $\Omega_S \subseteq (Q \times D)^\omega$. A finite computation is a finite prefix $\lambda = q_0 \xrightarrow{m_0} q_1 \xrightarrow{m_1} q_2 \ldots \xrightarrow{m_{n-1}} q_n \in (Q \times D)^* Q$ of some infinite sequence in Ω_S. We denote the set of all finite computations by Ω_S^+. For convenience, $(Q \times D)^\omega$ and $(Q \times D)^* Q$ shall be written as $(QD)^\omega$ and $(QD)^* Q$. The length of a computation λ, denoted by $|\lambda|$, is defined as the number of transitions in λ. For a finite computation $\lambda = q_0 \xrightarrow{m_0} q_1 \xrightarrow{m_1} q_2 \ldots \xrightarrow{m_{n-1}} q_n \in \Omega_S^+$, $|\lambda| = n$; for an infinite computation $\lambda \in q_0 \xrightarrow{m_0} q_1 \xrightarrow{m_1} \ldots \Omega_S$, $|\lambda| = \infty$. Given a computation $\lambda \in \Omega_S^{(+)}$, $\lambda(i) = q_i$ for all $i \in \{0, \ldots, |\lambda|\}$; $\lambda(i, j) = q_i \ldots q_j$ for all $i, j \in \{0, \ldots, |\lambda|\}$ and $i \leq j$; $m_\lambda = m_0 m_1 \ldots$ as the projection of actions in λ and $m_\lambda(i) = m_i$ for $i \in \{0, \ldots, |\lambda| - 1\}$. Note that $\lambda(|\lambda|)$ is the last state in λ. Finally, $\Omega_{S,q}^{(+)} = \{\lambda \in \Omega_S^{(+)} \mid \lambda(0) = q\}$ denotes the set of (finite) computations starting from $q \in Q$. Given a finite computation $\lambda \in \Omega_S^+$ and a coalition A, the cost of joint actions by A is defined as $cost_A(\lambda) = \sum_{i=0}^{|\lambda|-1} cost(\lambda(i), m_\lambda(i))$.

Definition 2.2. Given a pRCGS S, a strategy of an agent $a \in N$ is a mapping $f_a : (QD)^* Q \to \mathcal{D}(\mathbb{N}^+)$ which associates each sequence $\lambda \in (QD)^* Q$ to a distribution $\mu_a \in \mathcal{D}(\{1, \ldots, d(\lambda(|\lambda|), a)\})$.

Definition 2.3. A strategy is called *memoryless* (or Markovian) if its choice of actions depend only on the current state, i.e., $f_a(\lambda) = f_a(\lambda(|\lambda|))$ for all $\lambda \in (QD)^* Q$. It is called *deterministic* if it always selects a action with probability 1, i.e., $f_a(\lambda)$ is a Dirac distribution.

Definition 2.4. Given a pRCGS S, a coalition strategy $F_A : A \to ((QD)^* Q \to \mathcal{D}(\mathbb{N}^+))$ is a function which associates each agent a in A with a strategy.

Given a coalition stratgegy F_A, each finite sequence $\lambda \in (QD)^* Q$ gives rise to a distribution $\mu_\lambda^{F_A} \in \mathcal{D}(A \to D_A(\lambda(|\lambda|)))$ over joint actions $m \in D_A(\lambda(|\lambda|))$ where $\mu_\lambda^{F_A}(m) = \prod_{a \in A} f_a(\lambda)(m(a))$ and $f_a = F_A(a)$ for all $a \in A$. Given two coalition strategies F_A and F_B of two disjoint coalitions A and B, i.e., $A \cap B = \emptyset$, their union is also a coalition strategy, denoted by $F_A \cup F_B$, for $A \cup B$.

Definition 2.5. Given a bound $b \in \mathbb{B}$, $k \in \mathbb{N}$, and a strategy F_A, F_A is b-bounded iff for all $\lambda \in \Omega_S^+$, it holds that $cost_A(\lambda) \leq b$ and $\text{supp}(\mu_\lambda^{F_A}) \subseteq \{m \in D_A(\lambda(|\lambda|)) \mid cost(\lambda(|\lambda|), m) \leq b - cost(\lambda)\}$.

Given a state $q_0 \in Q$, we can determine the probability of every finite computation $\lambda = q_0 \xrightarrow{m_0} q_1 \xrightarrow{m_1} q_2 \ldots \xrightarrow{m_{n-1}} q_n \in \Omega_{S,q}^+$ consistent with F_A as $\text{Pr}_{S,q_0}^{F_N}(\lambda) = \prod_{i=0}^{n-1} \mu_{\lambda(0,i)}^{F_N}(m_i) \times \delta(s_i, m_i)(s_{i+1})$. For each finite computation $\lambda \in \Omega_S^+$, we can then define a cylinder set C_λ that consists of all infinite computations prefixed by λ. Given an initial state $q \in Q$, it is then standard [4, 11] to define a measurable space over $\Omega_{S,q}$, infinite runs of S from q, as $(\Omega_{S,q}, \mathcal{F}_{S,q})$ where $\mathcal{F}_{S,q} \subseteq \wp(\Omega_{S,q})$ is the least σ-algebra on $\Omega_{S,q}$ generated by the family of all cylinder sets C_λ where $\lambda \in \Omega_S^+$ that starts from q, i.e., $\lambda(0) = q$. Given a strategy F_N, a strategy for all agents in the game, the behaviour of S is fully probabilistic. It then gives rise to a probability measure $(\Omega_{S,q}, \mathcal{F}_{S,q}, \text{Pr}_{S,q}^{F_N})$ where $\text{Pr}_{S,q}^{F_N} : \mathcal{F}_{S,q} \to [0, 1]$ uniquely extends $\text{Pr}_{S,q}^{F_N} : \Omega_{S,q}^+ \to [0, 1]$ such that $\text{Pr}_{S,q}^{F_N}(C_\lambda) = \text{Pr}_{S,q}^{F_N}(\lambda)$ for all $\lambda \in \Omega_{S,q}^+$ starting from q.

2.3 Truth definition for pRB-ATL

Given a pRCGS $S = (n, r, Q, \Pi, \pi, d, c, \delta)$, the truth definition for pRB-ATL is given inductively as follows:

- $S, q \models \top$;
- $S, q \models p$ iff $q \in \pi(p)$;
- $S, q \models \neg\varphi$ iff $S, q \not\models \varphi$;
- $S, q \models \varphi_1 \vee \varphi_2$ iff $S, q \models \varphi_1$ or $S, q \models \varphi_2$;
- $S, q \models \langle\langle A^b \rangle\rangle \text{P}_{\bowtie v}[\psi]$ iff $\exists$ b-bounded F_A such that $\forall F_{\bar{A}}$, $\text{Pr}_{S,q}^{F_A \cup F_{\bar{A}}}(\{\lambda \in \Omega_{S,q} \mid S, \lambda \models \psi\}) \bowtie v$;
- $S, \lambda \models \bigcirc\varphi$ iff $S, \lambda(1) \models \varphi$;
- $S, \lambda \models \varphi_1 \, \mathcal{U}^k \, \varphi_2$ iff $\exists i \in \mathbb{N}$ such that $i \leq k$, $\forall j < i : S, \lambda(j) \models \varphi_1$, and $S, \lambda(i) \models \varphi_2$;
- $S, \lambda \models \neg\psi$ iff $S, \lambda \not\models \psi$.

3 CONCLUSIONS

The logic pRB-ATL proposed in this paper can be used to express various interesting properties of coalitions of agents involving resource limitations and probabilistic behaviour. We have also developed a standard model-checking algorithm for pRB-ATL. However, we are unable to present it due to space constraints.

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