Contingent Payment Mechanisms for Resource Utilization

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ABSTRACT

We introduce the problem of assigning resources to improve their utilization, for settings where agents have uncertainty about their own values for using a resource, and where it is in the interest of the society or the planner that resources be used and not wasted. Done in the right way, improved utilization maximizes social welfare—balancing the utility of a high value but unreliable agent with the group's preference that resources be used. We introduce the family of *contingent payment mechanisms* (CP), which may charge an agent contingent on use (a penalty). A CP mechanism is parameterized by a maximum penalty, and has a simple dominant-strategy equilibrium. Under a set of axiomatic properties, we establish welfare-optimality for the special case CP(W), with CP instantiated for a maximum penalty equal to societal value W for utilization. The special case with no upper bound on penalty, the *contingent second-price mechanism*, maximizes utilization. We extend the mechanisms to assign multiple, heterogeneous resources, and present a simulation study of the welfare properties of these mechanisms.

KEYWORDS

mechanism design; resource utilization; uncertainty

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1 INTRODUCTION

Allocated resources often go to waste, even when in scarce supply. It is common in university departments, for example, to find that all rooms are fully booked in advance, yet walking down the corridor one sees that many rooms are in fact empty. For another university related example, one of the authors of the present paper received the following email from the office of career services:

```
SITE VISIT: XXXXX Corporation
Date: Wednesday, January 17, 2018, 9am to 1pm
NUMBER OF PARTICIPANTS: 25 spots
RESERVATIONS: Reservations are now open. Reserve
your spot today!
COST: $15 to hold your reservation. There is no
charge for the visit. You will only be charged if
you cancel within a week before the trip or do
not show up on the morning of the visit.
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Another example considers conferences and symposiums where the organizers decide not to charge any fee for registration: all seats are usually taken within a few days after the registration is open, but it is not surprising to find that a large portion of badges made for participants who did get seats were never picked up until the end, despite the fact that many people were placed on the wait list and ultimately turned away. For examples from other domains, consider allocating spots in a spinning class (and other facilities e.g. squash courts) to members of a gym, and assigning time slots for a public electric vehicle charging station to residents in a neighborhood. Even a gym member who is highly uncertain about being able/willing to attend the class, or a resident unsure about actually needing to use the charging station, may reserve a space just in case this turns out to be convenient.

What is common to these problems is the presence of uncertainty, self-interest and down-stream utilization decisions on the part of participants, together with the broader interest of the society or the planner (a university, a corporation, the conference organizers, or the citizens of a city) that a resource be used and not wasted: utilization often has positive externalities beyond the immediate agents, e.g. the firm benefiting from potentially hiring more students if more showed up for the site visit; the planner might also be interested only in the utilization of the resources, e.g. conditioned on utilization, a school may not have preference over students with higher vs. lower willingness to pay for the squash courts.

We formalize the desire for utilization as a welfare gain of $W \geq 0$ when a resource is used, and adopt the design objective of maximizing expected social welfare: the sum of the expected value to the assigned agent and the expected value to society or the planner. The societal value W models either the positive externality on the society from utilization, or the weight assigned to the planner while trading-off agents' vs. planner's welfare. In the special case where $W = \infty$, the goal becomes one of maximizing utilization—the probability that the resource is used.

Despite being important to practice and simple to state, this problem does not appear to have been formally defined or studied in the literature. Running a second-price (SP) auction need not assign a resource to a reliable agent (the agent with the highest expected value for the option of using a resource need not be the one most likely to show up). Moreover, the SP auction does not

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charge payments contingent on whether or not the resource is used, therefore misses the opportunity to "shape" incentives to use a resource once it has been assigned. A penalty of $15 changes the calculus for an assigned agent: now she will choose to use the resource as long as her realized value is greater than -$15.

Beyond our opening examples, penalties for not using a resource are used by some hospitals in charging patients for missing appointments, by hiking clubs that charge their members for not showing up for trips they signed-up for, and by some restaurants who charge a fee if guests reserve tables but do not show up. These approaches can be viewed as simple, first-come first-served schemes, and where it is not clear how the penalty should be set: a penalty that is too small is not effective, whereas a penalty that is too big will drive away participation in the scheme. We are not aware of any formal analysis of these kinds of mechanisms, or a design approach that takes into account the penalty that an individual participant would be willing to face, which in fact is a good signal for her reliability. This is the main conceptual contribution of our paper.

Our Results. We formalize the problem of designing mechanisms for improving resource utilization, and introduce a family of two-period mechanisms that make use of payments that are contingent on whether or not a resource is used. In our model, an agent's private type corresponds to a distribution on her future value for using the resource— this value models her utility from using the resource minus the utility from her outside option, and as a result may be negative. In period 0, agents make reports with knowledge of their types. A mechanism assigns the resource, and may both collect a payment at this time as well as determine a penalty for the assigned agent in the event the resource is not used. In period 1, the assigned agent's value is (privately) realized, and the agent then decides whether or not to use the resource. We insist on voluntary participation, and also the mechanism being no-deficit, thus precluding charging very large penalties while also paying the agents a very large reward to participate in the first place.[1]

We introduce the class of *contingent payment mechanisms* (CP), parameterized by a maximum penalty Z. The CP mechanism has a simple dominant-strategy equilibrium (Theorem 3.3). The main results establish the welfare-optimality of the CP mechanism when instantiated for a maximum penalty equal to the societal value W for utilization, and under a set of axioms. First, CP(W) is not dominated for expected welfare by any other mechanism (Theorem 4.5). Second, amongst mechanisms that always allocate the resource and support a simple indirect structure, CP(W) is *ex-post* optimal, i.e. maximizes social welfare profile by profile (Theorem 4.7). As an interesting, and practically-motivated special case, the *contingent second-price* (CSP) mechanism is the special case of the CP mechanism with no upper bound on penalty. The CSP mechanism optimizes utilization (under the same assumptions), and also has the appealing property that it never collects a payment from an agent who uses the resource. We extend the mechanisms to the setting of multiple, heterogeneous resources (where each agent gets at most one resource) in Section 5,[2] and present simulation results in Section 6 to demonstrate the effectiveness of our mechanisms, comparing with second-price auctions and other benchmarks. The proofs of most results presented in this paper, as well as additional simulation results, are provided in a full version of this paper [16].

1.1 Related Work

Contingent payments have arisen in previous work. Prominent examples include auctioning oil drilling licenses [14], royalties [6, 9], ad auctions [24], and selling a firm [11]. Unlike in our model, payments are contingent on some observable world states (e.g. amount of oil produced, a click, or the ex post cash flow) rather than an agent's own downstream actions. Moreover, the major role of contingent payments in these applications is to improve revenue as well as to hedge risk [22]. In contrast, the role of penalties in our setting is two-fold: to provide participants with a way to signal their own, idiosyncratic uncertainty, as well as to address problems of moral hazard that arise once a resource has been assigned.

Our problem is a principal-agent problem [13, 15]. Classically, problems with hidden information before the time of contracting are termed *adverse selection*, and problems for which information asymmetry arises after contracting are referred to as *moral hazard* (see [4, 23]). The distinction between the two is blurred in dynamic settings such as the present one, and in particular there are informational asymmetries both before and after contracting. Although agents' actions are observable, uncertainty together with participation constraints precludes charging unbounded penalties, which is a standard approach when actions are observable in moral hazard.

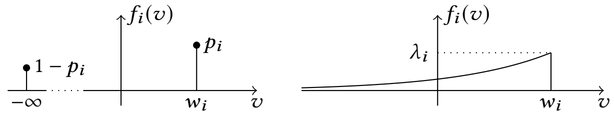
In regard to auctions in which actions take place after contracting, Atakan and Ekmecki [2] study auctions where the value of taking each action depends on the collective actions by others, but these actions are taken before rather than after observing the world state, and thus the timing of information is quite different than in our model. Courty and Li [8] study the problem of selling airline tickets, where passengers have uncertainty about their value at the time of booking, and decide whether to take a trip only after realizing their values. Although agents' types are modeled as distributions, and the optimal mechanism can be understood as a menu of contingent contracts, they consider distributions that satisfy either mean-preserving spread or stochastic dominance, and effectively reduce the type space to one-dimensional. This, in addition to their focus on revenue, is a departure from the present work.

The closest related work is on the design of mechanisms for incentivizing reliability in demand-side response in electric power systems [17–19], where selected agents decide whether to respond only after uncertainty in their costs are resolved. The objective there is to guarantee a probabilistic target on the collective actions taken by agents, and no optimality results are provided. Crucially, there is no hard feasibility constraint in these settings— that is, whereas only one agent can be assigned to a resource in our model, in demand response problems any number of agents can be selected.

Other papers study assignment problems under uncertainty, including models with the possibility that workers assigned to tasks will prove to be unreliable [21], and general models of dynamic mechanism design, where the goal is to maximize expected total

[1] Without the requirement of no-deficit, a simple second price auction for "the option to use a resource and also get paid W when the resource is used" is welfare-optimal.

[2] For assigning multiple heterogeneous resources, the generalized CP mechanisms are dominant-strategy incentive-compatible, however, the optimality results do not generalize, and we can construct examples to show that the VCG mechanism can achieve better expected welfare or utilization. Still, simulation results demonstrate significantly better performance on average.



(a) (w_i, p_i) model **(b) Exponential model**

Figure 1: Agent value distributions under two type models.

(discounted) value in the presence of uncertainty [3, 5, 20] (dynamic VCG mechanism). The difference between these models and our problem is that there is no need or possibility for the "shaping" of downstream behavior through contingent payments. In Porter et al. [21], for example, the probability that a worker fails to complete a task is fixed— this corresponds to the special case we discuss in Example 2.1, and we allow for much more general agent types. In dynamic VCG, the solution would be to simply run a second-price auction with reserve $-W$ in period 1 (i.e. allowing a payment of up to W to agents). This is outside our design space: we seek mechanisms that assign the resource in a period *before* the value of agents are realized, which is important in the aforementioned motivating settings, because it allows for planning by agents.

2 PRELIMINARIES

We first introduce the model for the assignment of a single resource. There is a set of agents $N = \{1, 2, \ldots, n\}$ and two time periods. In period $T = 0$, the value of each agent $i \in N$ for using the resource is uncertain, represented by a random variable $V_i \in \mathbb{R}$, whose exact (and potentially negative) value is not realized until period $T = 1$. The cumulative distribution function (CDF) F_i of V_i is agent i's private information at period 0, and corresponds to her *type*. Let $F = (F_1, \ldots, F_n)$ denote a type profile.

The assignment is determined in period 0, whereas the allocated agent decides on whether to use the resource at period 1, after she privately learns the realization v_i of V_i. In addition, if the resource is utilized, then society gains value $W \geq 0$. Define $V_i^+ \triangleq \max\{V_i, 0\}$. We make the following assumptions about F_i for each $i \in N$:

(A1) $\mathbb{E}\left[V_i^+\right] > 0$, which means that V_i takes positive value with non-zero probability, thus the *option* to use the resource as one wishes has positive value. An agent for which this is violated would never be interested in the resource.

(A2) $\mathbb{E}\left[V_i^+\right] < +\infty$, which means that agents do not get infinite expected utility from the option to use the resource.

The following are two example value distributions.

Example 2.1 ((w_i, p_i) model). The value for agent i to use the resource is $w_i > 0$, however, she is able to do so only with probability (w.p.) $p_i \in (0, 1)$. With probability $1 - p_i$, agent i is unable to use the resource. This hard constraint can be modeled as V_i taking value $-\infty$ with probability $1 - p_i$. See Figure 1a. We have $\mathbb{E}\left[V_i^+\right] = w_i p_i > 0$. If the resource is allocated to agent i, it will be used with probability p_i, and the expected social welfare is $p_i(w_i + W)$. □

Example 2.2 (Exponential model). The utility for agent i to use the resource is a fixed value $w_i > 0$ minus a random opportunity cost, which is exponentially distributed with parameter $\lambda_i > 0$. The probability density function of V_i is given by $f_i(v) = \lambda_i e^{\lambda_i(v - w_i)}$ for $v \leq w_j$, and $f_i(v) = 0$ for $v > w_i$. See Figure 1b. □

2.1 Two-Period Mechanisms

A *two-period mechanism* is defined by $\mathcal{M} = (\mathcal{R}, x, t^{(0)}, t^{(1)})$. At period 0, based on knowledge of her type F_i, each agent makes a report r_i from some set of messages $\mathcal{R}$. $r = (r_1, \ldots, r_n) \in \mathcal{R}^n$ denotes a report profile. Based on the reports, an *allocation rule* $x = (x_1, \ldots, x_n) : \mathcal{R}^n \to \{0, 1\}^n$ assigns the right to use the resource to at most one agent, which we denote as i^*, for whom $x_{i^*}(r) = 1$. $x_i(r) = 0$ for all $i \neq i^*$. Each agent is charged $t_i^{(0)}(r)$ in period 0. The mechanism also determines the penalty $t_{i^*}^{(1)}(r)$ for agent i^*.

At the beginning of period 1, the allocated agent privately observes the realized value v_{i^*} of V_{i^*}, and decides on whether to use the resource. The mechanism then collects the penalty $t_{i^*}^{(1)}(r)$ from i^* if she did not use the resource (let $t_i^{(1)}(r) = 0$ for $i \neq i^*$).[3]

Second Price and Contingent Second Price mechanisms. For example, the standard second price (SP) auction can be described as a two-period mechanism where $\mathcal{R} = \mathbb{R}_{\geq 0}$, $i^* \in \arg\max_{i \in N} r_i$, $t_{i^*}^{(0)}(r) = \max_{i \neq i^*} r_i$, and all other payments are 0. The SP auction does not make use of the period 1 payments. Another mechanism is the *contingent second price* (CSP) mechanism, which is the same as SP, except $t_{i^*}^{(1)}(r) = \max_{i \neq i^*} r_i$ instead of $t_{i^*}^{(0)}(r)$: the winner pays the second highest bid, *but only if she fails to use the resource.*

We assume that agents are risk-neutral, expected-utility maximizers with quasi-linear utility functions. Assume agent i is allocated the resource and is facing a *two-part payment* (z, y), where z is the period 1 *penalty* payment and y is the period 0 *base payment*. Her utility from using the resource in period 1 is $v_i - y$, and her utility from not using the resource is $-z - y$. Therefore, after observing v_i in period 1, the rational decision is to use the resource if and only if $v_i - y \geq -y - z \Leftrightarrow v_i \geq -z$ (breaking ties in favor of using the resource). Let $\mathbb{1}\{\cdot\}$ be the indicator function, and define $u_i(z)$ as

$$u_i(z) \triangleq \mathbb{E}\left[V_i \mathbb{1}\{V_i \geq -z\}\right] - z\mathbb{P}\left[V_i < -z\right] = \mathbb{E}\left[\max\{V_i, -z\}\right], \quad (1)$$

we know that the expected utility of an allocated agent facing two-part payment (z, y) is $u(z) - y$. Given report profile r, an agent's expected utility is therefore: $x_i(r) u_i(t_i^{(1)}(r)) - t_i^{(0)}(r)$.

Assume throughout the paper that the agents make rational decisions in period 1, if allocated. The interesting question is in agents' incentives regarding their reports in period 0. For any vector $s = (s_1, \ldots, s_n)$ and $i \in N$, we denote $s_{-i} \triangleq (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$.

Definition 2.3 (Dominant strategy equilibrium). A two-period mechanism has a *dominant strategy equilibrium* (DSE) if $\forall i \in N$, for any type F_i satisfying (A1) and (A2), there exists a report $r_i^* \in \mathcal{R}$ s.t. $\forall r_i \in \mathcal{R}$, $\forall r_{-i} \in \mathcal{R}^{n-1}$, $x_i(r_i^*, r_{-i}) u_i(t_i^{(1)}(r_i^*, r_{-i})) - t_i^{(0)}(r_i^*, r_{-i}) \geq x_i(r_i, r_{-i}) u_i(t_i^{(1)}(r_i, r_{-i})) - t_i^{(0)}(r_i, r_{-i})$.

Let $r^*(F) = (r_1^*, \ldots, r_n^*)$ denote the report profile under a DSE given type profile F.

Definition 2.4 (Individual rationality). A two-period mechanism is *individually rational* (IR) if $\forall i \in N$, for any F_i satisfying (A1) and (A2), $\forall r_{-i} \in \mathcal{R}^{n-1}$, $x_i(r_i^*, r_{-i}) u_i(t_i^{(1)}(r_i^*, r_{-i})) - t_i^{(0)}(r_i^*, r_{-i}) \geq 0$.

[3] More generally, we may think of mechanisms that charge the allocated agent a non-zero payment in period 1 even if she used the resource. Without temporal preference for money, it is without loss to move this part of the payment to period 0, and at the same time subtract the same amount from the penalty payment.

IR requires that an agent's expected utility is non-negative under her dominant strategy given that she makes rational decisions in period 1 (if allocated), regardless of the reports made by the rest of the agents. IR is based on the expected utility before uncertainty is resolved. It is still possible for an agent to get negative utility at the end of period 1. We cannot charge unallocated agents without violating IR, thus $t_i^{(0)}(r) \leq 0$ for all $i \neq i^*$ for all $r \in \mathcal{R}^n$.

The expected revenue of a two-period mechanism $\mathcal{M}$ is the total expected payment from the agents to the mechanism in DSE:

$$rev_{\mathcal{M}}(F) \triangleq \sum_{i \in N} t_i^{(0)}(r^*) + t_{i^*}^{(1)}(r^*) \cdot \mathbb{P}\left[V_{i^*} < -t_{i^*}^{(1)}(r^*)\right]. \quad (2)$$

Definition 2.5 (No deficit). A two-period mechanism satisfies *no deficit* (ND) if, for any type profile F that satisfies (A1) and (A2), the expected revenue is non-negative: $rev_{\mathcal{M}}(F) \geq 0$.

A mechanism is *anonymous* if the outcome is invariant to permuting the identities of agents. A mechanism satisfies *no subsidy* if the mechanism does not make payments to unallocated agents.

The *utilization* achieved by mechanism $\mathcal{M}$ is the probability with which the allocated agent rationally decides to use the resource:

$$ut_{\mathcal{M}}(F) \triangleq \mathbb{P}\left[V_{i^*} \geq -t_{i^*}^{(1)}(r^*)\right]. \quad (3)$$

The expected value to society from the resource being utilized is therefore $ut_{\mathcal{M}}(F)W$, and the expected *social welfare* is the sum of the values to the society and the agent from using this resource:

$$sw_{\mathcal{M}}(F) \triangleq \mathbb{E}\left[V_{i^*}\mathbb{1}\{V_{i^*} \geq -t_{i^*}^{(1)}(r^*)\}\right] + ut_{\mathcal{M}}(F)W. \quad (4)$$

Our objective is to design mechanisms that maximize expected social welfare. We do not consider monetary transfers in the social welfare function. The reason $t_i^{(1)}(r^*)$ appears is that it affects the decision of the allocated agent in period 1.

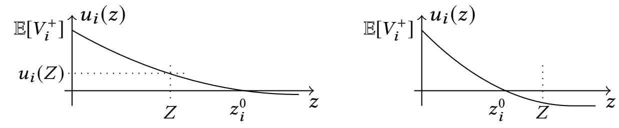
3 CONTINGENT PAYMENT MECHANISM

We introduce in this section a class of contingent payment mechanisms parametrized by a maximum penalty Z an agent may be charged in period 1, and show that under (A1) and (A2), the contingent payment mechanism with $Z = W$ achieves higher welfare and utilization than the second price auction in DSE. The uniqueness and optimality are discussed in Section 4.

Definition 3.1 (Contingent payment mechanism). The *contingent payment mechanism* with max. penalty Z (CP(Z)) collects two-part bids $b = (b_1, \ldots, b_n)$. For each $i \in N$, $b_i = (b_i^{(1)}, b_i^{(0)}) \in \mathcal{R}$, where $\mathcal{R} = \{(z, y) \in \mathbb{R}^2 \mid 0 \leq z \leq Z,\ y = 0\} \cup \{(z, y) \in \mathbb{R}^2 \mid z = Z,\ y \geq 0\}$.

- Allocation rule: $x_{i^*}(b) = 1$ for $i^* \in \arg\max_{i \in N}\{b_i^{(0)} + b_i^{(1)}\}$ (breaking ties at random); $x_i(b) = 0$ for all $i \neq i^*$,
- Payment rule: let $i' \in \arg\max_{i \neq i^*}\{b_i^{(0)} + b_i^{(1)}\}$. $t_{i^*}^{(0)}(b) = b_{i'}^{(0)}$; $t_{i^*}^{(1)}(b) = b_{i'}^{(1)}$; $t_i^{(0)}(b) = t_i^{(1)}(b) = 0$, for all $i \neq i^*$.

Under the CP(Z) mechanism, each agent may bid a period 0 base payment if she is willing to bid Z as penalty, in which case $b_i = (Z, b_i^{(0)})$ for some $b_i^{(0)} \geq 0$. Otherwise, she may bid a maximum acceptable penalty (up to Z) and no base payment, i.e. $b_i = (b_i^{(1)}, 0)$ for some $b_i^{(1)} \in [0, Z]$. The resource is allocated to the highest base payment bidder, if there exist agents with $b_i^{(0)} > 0$ (since $b_i^{(1)} \leq Z$ thus $b_i^{(1)} + 0 \leq b_i^{(0)} + Z$). Otherwise, the resource is allocated to the highest penalty bidder. The allocated agent is charged a two-part payment equal to the bid of the second "highest" bidder. When $Z = 0$, we get the SP auction, as agents cannot bid on the penalty.



(a) $u_i(Z) \geq 0$ (b) $u_i(Z) < 0$

Figure 2: Expected utility as a function of penalty z.

Recall that $u_i(z)$ as defined in (1) is the expected utility of agent i if she is allocated and charged only a penalty z. $u_i(z)$, as a result, is also the highest period 0 base payment agent i is willing to accept, when her penalty is z. We first state some properties that are useful for establishing the DSE under CP(Z).

Lemma 3.2. *Assuming (A2), the expected utility $u_i(z)$ as a function of the penalty z satisfies:*
(i) $u_i(0) = \mathbb{E}\left[V_i^+\right]$, $\lim_{z \to +\infty} u_i(z) = \mathbb{E}\left[V_i\right]$, and
(ii) $u_i(z)$ is continuous, convex, and monotonically decreasing in z.

See Figure 2. For part (i), $u_i(0) = \mathbb{E}[V_i^+]$ by definition, and $\lim_{z \to \infty} \mathbb{E}\left[u_i(z)\right] = \mathbb{E}\left[V_i\right]$ holds by the monotone convergence theorem. Part (ii) holds since $\max\{V_i, -z\}$ is monotonically decreasing in z, continuous, and convex, so $u_i(z)$ inherits these properties. Intuitively, when $z = 0$, the agent uses the resource iff the realized value is non-negative, thus gets expected utility $\mathbb{E}[V_i^+]$. As z increases, the expected utility continuously decreases. When $z = \infty$, the agent always uses the resource and never pays the penalty, thus her expected utility converges to $\mathbb{E}\left[V_i\right]$.

Theorem 3.3 (Dominant strategy in CP(Z)). *Given (A1)-(A2), under the contingent payment mechanism with maximum penalty Z, it is a dominant strategy for each agent $i \in N$ to bid $b_{i,\text{CP}}^* = (Z, u_i(Z))$ if $u_i(Z) \geq 0$. Otherwise, it is a dominant strategy to bid $b_{i,\text{CP}}^* = (z_i^0, 0)$, where z_i^0 is the unique zero-crossing of $u_i(z)$.*

Proof. First, observe that the message space $\mathcal{R}$ is effectively one-dimensional. For any $(b_i^{(1)}, b_i^{(0)})$, $(\tilde{b}_i^{(1)}, \tilde{b}_i^{(0)}) \in \mathcal{R}$, denote $(b_i^{(1)}, b_i^{(0)}) \geq (\tilde{b}_i^{(1)}, \tilde{b}_i^{(0)})$ if $b_i^{(1)} + b_i^{(0)} \geq \tilde{b}_i^{(1)} + \tilde{b}_i^{(0)}$. For any agent type F_i under (A1) and (A2), the agent's expected utility is weakly lower for a higher payment, i.e. $b_i^{(1)} + b_i^{(0)} \geq \tilde{b}_i^{(1)} + \tilde{b}_i^{(0)} \Rightarrow u_i(b_i^{(1)}) - b_i^{(0)} \leq u_i(\tilde{b}_i^{(1)}) - \tilde{b}_i^{(0)}$.

We now show that for any agent, her expected utility at the two-part bid $b_{i,\text{CP}}^*$ is exactly zero. If $u_i(Z) \geq 0$, agent i gets expected utility $u_i(Z) - u_i(Z) = 0$ if she is charged $b_{i,\text{CP}}^* = (Z, u_i(Z))$. In the case where $u_i(Z) < 0$, the continuity, convexity and monotonicity of $u_i(z)$ (part (ii) of Lemma 3.2) implies that there is a unique zero crossing $z_i^0 < Z$ of $u_i(z)$ s.t. $u_i(z_i^0) = 0$. If agent i is charged $b_{i,\text{CP}}^* = (z_i^0, 0)$, her expected utility is then $u_i(z_i^0) - 0 = 0$. This implies that $b_{i,\text{CP}}^*$ is an agent's "highest acceptable payment" in the $\mathcal{R}$. The argument for dominant strategy is then standard, observing that the mechanism allocates to the "highest" bidder, and charges the allocated agent a second "highest" bid. □

When $Z = 0$, the CP(Z) mechanism reduces to the SP auction, where it is a DSE to bid $b_{i,\text{SP}}^* = \mathbb{E}[V_i^+]$. When $Z \to +\infty$, and with

(A3) $\mathbb{E}[V_i] < 0$,

then CP(Z) reduces to the CSP mechanism, where it is a dominant strategy for each agent i to bid her largest acceptable penalty z_i^0, which the unique zero crossing of $u_i(z)$ (see Figure 2). z_i^0 exists and is unique given (A3), since $u_i(z)$ is continuous, monotonically decreasing in z, and converges to $\mathbb{E}[V_i] < 0$.[4]

3.1 Better Welfare and Utilization than SP

The following lemmas states useful properties of utilization and expected social welfare as functions of penalty z.

LEMMA 3.4. *Assuming (A1) and (A2), when agent i is allocated and charged a two-part payment (z, y), the utilization and expected social welfare are independent of the base payment y, and satisfy:*

(i) the utilization $ut_i(z) \triangleq \mathbb{P}[V_i \geq -z]$ is right continuous and monotonically non-decreasing in z. Moreover, $ut_i(z) = 1 + u_i'(z+)$, where $u_i'(z+)$ is the right derivative of u_i at z.

(ii) the social welfare $sw_i(z) \triangleq \mathbb{E}[V_i \mathbb{1}\{V_i \geq -z\}] + W\mathbb{P}[V_i \geq -z]$ is right continuous, monotonically non-decreasing in z when $z \leq W$, and monotonically non-increasing in z when $z > W$.

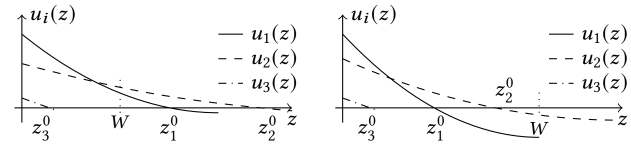
The continuity and monotonicity of $ut_i(z)$ are obvious. From Fubini's theorem and the fundamental theorem of calculus, we can show that the right derivative of $u_i(z)$ is equal to the right limit of $-F_i(-v)$ at z, which is equal to $\mathbb{P}[V_i \geq -z] - 1$. Intuitively, the agent uses the resource with higher probability when the penalty z increases. This, in turn, results in a smaller probability of paying the penalty, thus $u_i(z)$ decreases *slower* as z increases, corresponding to a shallower slope of the convex function $u_i(z)$. For part (ii), observe that $sw_i(z) = \mathbb{E}[(V_i + W)\mathbb{1}\{V_i \geq -z\}]$, and that $V_i + W$ is non-negative iff. $V_i \geq -W$, thus charging $z = W$ optimizes $sw_i(z)$.

LEMMA 3.5. *Let $u_1(z)$ and $u_2(z)$ be the expected utilities of two agents whose types satisfy (A1) and (A2), and consider $z_1, z_2 \in \mathbb{R}$ s.t. $z_1 < z_2$. If $u_1(z_1) \geq u_2(z_1)$, and $u_1(z_2) \leq u_2(z_2)$, we have: (i) $ut_1(z_1) \leq ut_2(z_2)$, and (ii) $sw_1(z_1) \leq sw_2(z_2)$ if $z_1 \leq z_2 \leq W$, and $sw_1(z_1) \geq sw_2(z_2)$ if $W \leq z_1 \leq z_2$.*

Intuitively, when $u_2(z)$ crosses $u_1(z)$ from below, $u_1(z_2)-u_1(z_1) \leq u_2(z_2) - u_2(z_1)$. The convexity of $u_i(z)$ then implies that the right derivative of $u_2(z)$ at z_2 must be higher than the right derivative of $u_1(z)$ at z_1, hence the inequality on utilization. The CP mechanism with the maximum penalty set to $Z = W$ will have some very nice optimality properties. As a preliminary observation, we state the following result relative to the SP auction.

THEOREM 3.6. *For any set of agent types satisfying (A1)-(A2), under the dominant strategy equilibria, the CP(W) mechanism Pareto-dominates the second price auction in utilization and social welfare.*

We leave the proof to the full paper [16]. When SP and CP(W) allocate the resource to the same agent, Lemma 3.4 implies that CP(W), charging a non-negative penalty, (weakly) improves utilization and welfare. In the case where SP and CP(W) allocate to agents 1 and 2 respectively, we have two cases as shown in Figure 3. Observe



(a) $u_2(W) \geq 0$ **(b)** $u_2(W) < 0$

Figure 3: Example economies where the SP winner (agent 1) is different from the CP(W) winner (agent 2.)

$$V_1 = \begin{cases} 100, & \text{w.p. } 0.2, \\ -20, & \text{w.p. } 0.4, \\ -\infty, & \text{w.p. } 0.4, \end{cases} \quad V_2 = \begin{cases} 40, & \text{w.p. } 0.4, \\ -10, & \text{w.p. } 0.4, \\ -\infty, & \text{w.p. } 0.2. \end{cases}$$

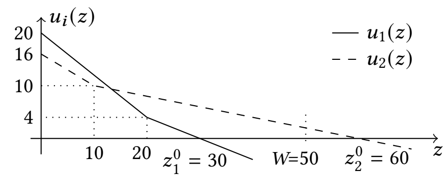


Figure 4: Agents' value distributions and expected utility functions in Example 3.7.

that $u_1(0) \geq u_2(0)$ and $u_2(\min\{z_1^0, W\}) \geq u_1(\min\{z_1^0, W\})$. Let z^* be the penalty that agent 2 is charged under CP(W), we also have $z^* \in [\min\{z_1^0, W\}, W]$. Lemmas 3.4 and 3.5 then imply $ut_2(z^*) \geq ut_2(\min\{z_1^0, W\}) \geq ut_1(0)$, $sw_2(z^*) \geq sw_2(\min\{z_1^0, W\}) \geq sw_1(0)$.

The domination result holds for arbitrary tie-breaking rules for the two mechanisms. The same analysis on CSP shows that it always achieves a higher utilization than SP. We illustrate through the following examples the improvement in welfare and utilization from CP(W) over SP, and show that SP can be arbitrarily worse.

Example 3.7 (Double gain in CP(W)). Consider $W = 50$, and two agents with value distributions and expected utilities as shown in Figure 4. Compared with agent 2, agent 1 has higher value for the resource, but lower probability of willing to use the resource and higher probability for a hard constraint. Under SP, the DSE bids are $b_{1,\text{SP}}^* = 20$, $b_{2,\text{SP}}^* = 16$ thus agent 1 is allocated. The utilization is $ut_1(0) = \mathbb{P}[V_1 \geq 0] = 0.2$ and the social welfare is $sw_1(0) = 100 * 0.2 + 50 * 0.2 = 30$. Whereas under CP($W$), $b_{1,\text{CP}}^* = (z_1^0, 0) = (30, 0)$ and $b_{2,\text{CP}}^* = (W, u_2(W)) = (50, 2)$. Agent 2 is allocated and charged penalty $t_2^{(1)}(b) = 30$, thus the utilization is $\mathbb{P}[V_2 \geq -30] = 0.8$, and the social welfare is $sw_2(30) = 40 * 0.4 - 10 * 0.4 + 50 * 0.8 = 52$. Note that these are higher than $ut_2(0) = \mathbb{P}[V_2 \geq 0] = 0.4$ and $sw_2(0) = 36$— what is achieved if agent 2 is allocated the resource under SP in some other economy, and charged no penalty. □

Example 3.8 (SP arbitrarily worse). Under the (w_i, p_i) model introduced in Example 2.1, the expected utility for agent i given penalty z is $u_i(z) = w_i p_i - (1-p_i)z$. Consider an economy with two (w_i, p_i) agents: $p_1 = \varepsilon$, $w_1 = 1/\varepsilon$, and $p_2 = 1-\epsilon$, $w_2 = 1$ for some small $\varepsilon > 0$. Agent 1 is allocated under SP since $b_{1,\text{SP}}^* = 1 > b_{2,\text{SP}}^* = 1 - \varepsilon$. Agent 2 is allocated under CP(W) as long as $W > 1$, since $z_1^0 = 1/(1-\varepsilon) \approx 1$ and $z_2^0 = (1-\varepsilon)/\varepsilon \gg 1$. The utilization under SP and CP(W) are ε and $1 - \varepsilon$, respectively, and the welfare under the two mechanisms are $1 + \varepsilon W$ and $(1-\varepsilon)(1 + W)$. Thus, CP(W) can have arbitrarily better utilization and welfare by selecting a better winner. □

[4]Note that the value of the *option* to use the resource is $\mathbb{E}[V_i^+]$, which we assume is positive. (A3) only requires that an agent gets negative expected utility from committing to *always* use the resource, regardless of what happens. This is natural: without (A3), an agent would accept any unboundedly large penalty for the right to use a resource.

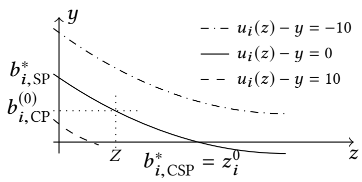


Figure 5: Iso-profit curves in the 2-D payment space.

The higher welfare and utilization achieved by the CP(W) mechanism come from two different aspects of its design. First, charging a penalty $z \in [0, W]$ changes the period 1 decision of the allocated agent, promoting the resource to be used more efficiently. Second, the CP(W) mechanism selects a better winner:

- For i s.t. $u_i(W) \geq 0$, the DSE two-part bids under CP(W) add up to $u_i(W) + W = \mathbb{E}[V_i \mathbb{1}\{V_i \geq W\}] - W\mathbb{P}[V_i < -W] + W = \mathbb{E}[(V_i + W)\mathbb{1}\{V_i \geq -W\}] = sw_i(W)$, the highest achievable welfare from allocating the resource to agent i and setting an optimal penalty $z = W$. As a result, if $\max_{i \in N}\{b_i^{(0)} + b_i^{(1)}\} \geq W$, CP($W$) selects the agent with highest achievable welfare.
- When W is large and $u_i(W) < 0$, agents with higher probabilities of showing up have $u_i(z)$ that decrease more slowly with z, thus have relatively higher zero crossing z_i^0, and are more likely to be allocated. With large W, higher utilization is more likely to generate higher social welfare.

4 CHARACTERIZATION AND OPTIMALITY

In this section, we study the optimal mechanism design problem with the following properties:[5]

P1. Dominant-strategy equilibrium	P4. Anonymous
P2. Individually rational	P5. Deterministic
P3. No deficit	P6. No subsidy

Recall that while facing a two part payment (z, y), an agent's expected utility is $u_i(z) - y$. We work with *iso-profit curves* in the two dimensional payment space, which are sets of (z, y) pairs for which $u_i(z) - y = \alpha$ for some constant α, i.e. an agent will be indifferent to all payments (z, y) that reside on the same iso-profit curve. See Figure 5. The *zero-profit curve* (i.e. where $\alpha = 0$, the solid line depicted in Figure 5) is characterized by $y = u_i(z)$, thus is continuous, convex, monotonically decreasing (Lemma 3.2).

4.1 Optimality of the CP(W) Mechanism

Define the *frontier* of a set of agents N with type profile F to be the upper-envelope of the zero-profit curves of all agents, i.e. for all $z \in \mathbb{R}$, $u_N(z) \triangleq \max_{i \in N} u_i(z)$. This characterizes the maximum willingness to pay (as base payment, given penalty z) by all agents in N. As the upper envelope of a finite set of continuous, convex, and decreasing functions, $u_N(z)$ has the same properties. When (A3) is satisfied by all agents, $u_N(z)$ also has a unique zero-crossing, which we denote as z_N^0. Define the frontier of the sub-economy without agent i as $u_{N\setminus\{i\}}(z) \triangleq \max_{j \neq i} u_j(z)$, and the m^{th} *frontier* of the economy as the m^{th} upper envelope of $\{u_i(z)\}_{i \in N}$. See Figure 6.

[5]For (P5) deterministic, we require that the outcome is deterministic unless multiple agents make the same reports, and that when breaking ties, the two-part payment each agent may be charged if allocated is still deterministic. (P5) also requires *positive responsiveness*, i.e. if a tied agent was to make a "strictly higher" report in an otherwise equivalent economy, then she has to be allocated with probability one in this other economy. See the full paper for a more detailed discussion.

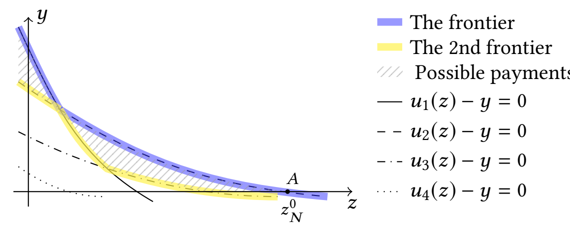


Figure 6: Characterization of possible outcomes under mechanisms that satisfy (P1)-(P6).

We first characterize the possible outcomes for any two-period mechanism satisfying (P1)-(P6) in the following lemma.

Lemma 4.1. *Assume that the type space includes all value distributions satisfying (A1)-(A3), and consider a two-period mechanism that satisfies (P1)-(P6). For any type profile F, the allocated agent i^* and the two-part payment (z^*, y^*) agent i^* is charged satisfy:*

(i) (z^*, y^*) resides weakly below $u_{i^*}(z)$.
(ii) (z^*, y^*) resides weakly above the frontier of the rest of the economy $u_{N\setminus\{i^*\}}(z)$.
(iii) The allocated agent faces a non-negative base payment $y^* \geq 0$.

Instead of requiring the type space to include all value distributions that satisfy (A1)-(A3), this lemma also holds assuming that the type space is the set of all (w_i, p_i) types. Part (i) is implied by IR. If part (ii) is violated, i.e. if there exists agent $i \neq i^*$ s.t. $u_i(z^*) - y^* > 0$, then in the economy where the type of agent i^* is also given by F_i, pretending that her type is F_{i^*} is a useful deviation. Part (iii) is proved by showing that if the allocated agent is charged $y^* < 0$ in some economy, we can replace the agent's type with some (w_i, p_i) type, in which case either IC or ND is violated.

Given (P4) anonymity, regardless of whether there are ties, there is a two-part payment (z^*, y^*) a mechanism charges its allocated agent(s). Lemma 4.1 implies that (z^*, y^*) is in between the first and second frontiers and above the horizontal axis (see Figure 6), and that the allocated agent(s) resides on the frontier at z^* (i.e. $u_N(z^*) = u_{i^*}(z^*)$ if agent i^* is allocated). The following lemma proves monotonicity of utilization and welfare w.r.t. the penalty z^*, achieved by mechanisms under (P1)-(P6), for any fixed economy.

Lemma 4.2. *Fix any type profile F satisfying (A1) and (A2). Among all mechanisms with (P1)-(P6), the utilization achieved by a mechanism is higher if it charges its allocated agent(s) a higher penalty z^*. Similarly, the achieved welfare is monotonically increasing in z^* when $z^* \leq W$, and monotonically decreasing in z^* when $z^* > W$.*

An important implication of this lemma is that the highest possible welfare achievable by any mechanism under (P1)-(P6) is achieved by charging a penalty $z^* = W$ if $\max_{i \in N} u_i(W) \geq 0$ and $z^* = \max_{i \in N} z_i^0$ otherwise. Lemma 4.1 then requires allocating to agent(s) in $\arg\max_{i \in N} u_i(z^*)$, which is in fact the set of agents allocated under CP(W). Therefore, the only ways to achieve an even higher welfare than the CP(W) mechanism are (i) break ties in favor of higher welfare instead of at random, and (ii) charge a higher penalty, when the CP(W) penalty determined by the second highest bid is lower than the optimal penalty z^*.

Definition 4.3 (Generic input). A type profile F satisfies the *generic input* property if for any $i, j \in N$, $i \neq j$: we have (i) $u_i(W) \neq u_j(W)$, if $u_i(W),\ u_j(W) \geq 0$, and (ii) $z_i^0 \neq z_j^0$, if $u_i(W),\ u_j(W) < 0$.

A type profile is generic if no two agents have the same period 0 willingness to pay given penalty W, or the same maximum acceptable penalty that is below W. As a result, there would not be any tie under the CP(W) mechanism.[6] An immediate result is that the CP mechanism is welfare optimal for the (w_i, p_i) type space with the generic input assumption, since in this type domain, a higher penalty does not improve utilization, or induce more welfare-optimal period 1 utilization decision of the allocated agent.

Corollary 4.4. *Assume the type space is the set of all (w_i, p_i) value distributions. With the generic input assumption, the CP(W) mechanism is welfare-optimal type profile by type profile among all two-period mechanisms that satisfy (P1)-(P6).*

We also have the following result, the first of our two main results. Theorem 4.5 states that the CP(W) mechanism is not dominated in welfare by any two-period mechanism under (P1)-(P6).

Theorem 4.5. *Assume the type space is the set of all value distributions satisfying (A1) and (A2). Assuming generic input, no two-period mechanism under (P1)-(P6) achieves weakly higher social welfare than the CP(W) mechanism for all type profiles, and a strictly higher social welfare than the CP(W) mechanism for at least one type profile.*

Intuitively, if a mechanism $\mathcal{M}$ under (P1)-(P6) always achieves weakly higher welfare than CP(W), lemmas 4.1 and 4.2 require that it always allocates the resource to the winner under CP(W). We then show a violation of either IR or DSE, if $\mathcal{M}$ ever charges the a higher penalty than the CP(W) mechanism does to improve welfare.

A payment space $\mathcal{P}$ of a mechanism with message space $\mathcal{R}$ is the set of two-part payments that's achievable by some report profile of the agents: $\mathcal{P} = \{(t_{i^*}^{(1)}(r), t_{i^*}^{(0)}(r)) \in \mathbb{R}^2 \mid r \in \mathcal{R}^n\}$.

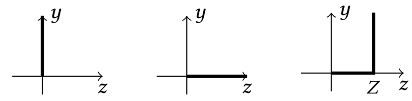
Definition 4.6 (Ordered payment space). A payment space $\mathcal{P} \subseteq \mathbb{R}^2$ is *ordered* if all agents with types satisfying (A1) and (A2) agree on which one of any two pairs of payments is more preferable. Formally, $\forall (z, y),\ (\tilde{z}, \tilde{y}) \in \mathcal{P}$, for all F_1, F_2 under (A1) and (A2),

$$u_1(z) - y > u_1(\tilde{z}) - \tilde{y} \Rightarrow u_2(z) - y \geq u_2(\tilde{z}) - \tilde{y}.$$

The second main result is that the CP(W) mechanism is welfare-optimal profile by profile among a large class of mechanisms that always allocate the resource, and use an ordered payment space.

Theorem 4.7. *Assume the type space is the set of all value distributions satisfying (A1) and (A2). With the generic input assumption, the CP(W) mechanism is welfare-optimal type profile by type profile, among all two-period mechanisms that satisfy (P1)-(P6), always allocate the resource, and use an ordered payment space.*

In fact, all mechanisms discussed so far use ordered payment spaces. The second price auction always charges no penalty and a non-negative base payment, thus $\mathcal{P}_{\text{SP}} = \{(z, y) \in \mathbb{R}^2 \mid z = 0,\ y \geq 0\}$, as illustrated in Figure 7a. Similarly, $\mathcal{P}_{\text{CSP}} = \{(z, y) \in \mathbb{R}^2 \mid z \geq 0,\ y = 0\}$, as in Figure 7b. The CP(Z) mechanism sets payments $\mathcal{P}_{\text{CP}(Z)} = \{(z, y) \in \mathbb{R}^2 \mid 0 \leq z \leq Z,\ y = 0\} \cup \{(z, y) \in \mathbb{R}^2 \mid z = Z,\ y \geq 0\}$, as illustrated in Figure 7c. In addition, mechanisms with an ordered payment space support a simple indirect message structure: the interpretation is that the mechanism asks an agent to report the largest payment in the ordering that is acceptable. Given this, dominant-strategy mechanisms can be achieved by allocating to the agent with the highest report and charging the second-highest report (defined with respect to the payment order).



(a) $\mathcal{P}_{\text{SP}}$ **(b)** $\mathcal{P}_{\text{CSP}}$ **(c)** $\mathcal{P}_{\text{CP}(Z)}$

Figure 7: Ordered payment spaces for various mechanisms.

[6] The generic inputs assumption is only needed for the indirect, CP(W) mechanism. A direct revelation version, that always breaks ties in favor of the agent with higher utilization, has all the performance guarantees stated in Corollary 4.4 and Theorems 4.5 and 4.7 without the generic input assumption.

4.2 Uniqueness and Optimality of CSP

The CSP mechanism can be considered as a special case of the CP(W) mechanism with no upper bound on the penalty ($Z = \infty$). Defining *generic input* as no ties in agents' maximum acceptable penalties, we can obtain the following analogous optimality results for utilization. Under the additional assumption of *no-charge* when using the resource, we can state a uniqueness result for CSP.

Theorem 4.8. *Assume the type space is the set of all value distributions satisfying (A1)-(A3), assume generic input, and consider two-period mechanisms that satisfy (P1)-(P6):*

(i) the CSP mechanism is the unique mechanism that always allocates the resource, and does not charge the allocated agent if the resource is utilized (i.e. "no-charge").
(ii) for the (w_i, p_i) type space, the CSP mechanism is optimal for utilization, type profile by type profile.
(iii) the CSP mechanism is not dominated for utilization.
(iv) the CSP mechanism is utilization optimal type profile by type profile, among all mechanisms that always allocate the resource and use an ordered payment space.

5 ASSIGNMENT OF MULTIPLE RESOURCES

In this section, we generalize the model to allow for assigning multiple, heterogeneous resources but where each agent remains interested in receiving at most one resource (i.e., the unit-demand setting). Let $N = \{1, 2, \ldots, n\}$ be the set of n agents and $M = \{a, b, \ldots, m\}$ be the set of m resources. For each $a \in M$, the value for each agent $i \in N$ to use resource a is a random variable $V_{i,a}$ with CDF $F_{i,a}$. $\{F_{i,a}\}_{a \in M}$ corresponds to agent i's type.

The SP auction can be generalized as the VCG mechanism [7, 12, 25], where it is a dominant strategy for agent i to bid $u_{i,a}(0) = \mathbb{E}[V_{i,a}^+]$ for each $a \in M$. The naive generalization of the CP(Z) mechanism fails to be incentive compatible, since agents' expected utilities are not quasi-linear in the period 1 penalty payments.

A set of two-part payments $\{(z_a, y_a)\}_{a \in M}$ is a set of *competitive equilibrium* (CE) price if the market clears when each agent selects her favorite resource given these payments: no resource is selected more than once, and a resource $a \in M$ that is not selected has zero prices: $z_a = y_a = 0$. Recall that in the message space of a CP(Z) mechanism, a two-part payment (z, y) is "higher" if it has a larger sum $z + y$, and that an agent has a lower expected utility if she is charged a higher two-part payment. We generalize the CP(Z) mechanism as the minimum CE price mechanism [1, 10]:

Definition 5.1 (Generalized CP(Z) mechanism). The generalized CP mechanism with max. penalty Z (the GCP(Z) mechanism) collects distributions $\{F_{i,a}\}_{i \in N, a \in M}$ from the agents, and computes the minimum CE payments $\{(z_a, y_a)\}_{a \in M}$ in the payment space $\mathcal{R} = \{(z, y) \in \mathbb{R}^2 \mid 0 \leq z \leq Z,\ y = 0\} \cup \{(z, y) \in \mathbb{R}^2 \mid z = Z,\ y \geq 0\}$.

- Allocation rule: allocate to each agent her favorite resource given the min CE payments: $x_i(F) = a_i^* \in \arg\max_{a \in M} u_{i,a}(z_a) - y_a$ (breaking ties to clear the market), if $\max_{a \in M} u_{i,a}(z_a) - y_a > 0$.
- Payment rule: charge each agent $t_i^{(1)}(F) = z_{a_i^*}$ and $t_i^{(0)}(F) = y_{a_i^*}$ if agent i is allocated resource a_i^*. All other payments are zero.

For the case where the m resources are identical, the mechanism reduces to the $(m + 1)^{\text{th}}$ price version of the CP(Z) mechanism. When $Z = 0$, the mechanism reduces to VCG, and for the case when $Z = \infty$, we get the generalized CSP (GCSP) mechanism, which prices each resource at the minimum CE penalties. Demange and Gale [10] prove that the minimum CE price mechanism is incentive compatible, and Alaei et al. [1] provide a recursive algorithm to compute these minimum CE prices.

Theorem 5.2. *Given assumptions (A1)-(A2), under the generalized CP(Z) and the generalized CSP mechanisms, it is a dominant strategy for each agent to truthfully report her type.*

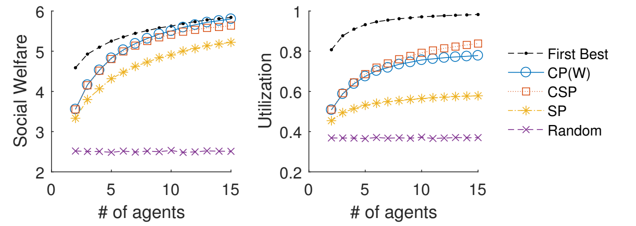
6 SIMULATION RESULTS

In this section, we compare the welfare and utilization achieved by different mechanisms. We adopt the exponential type model (see Example 2.2), under which agent i's value for using resource a is $V_{i,a} = w_{i,a} - O_{i,a}$, where $w_{i,a} > 0$ is the fixed value and $O_{i,a} \sim \text{Exp}(\lambda_{i,a})$ is the exponentially distributed opportunity cost. $\mathbb{E}\left[V_{i,a}\right] = w_{i,a} - \lambda_{i,a}^{-1}$ where $\lambda_{i,a}^{-1}$ is the expectation of $O_{i,a}$.

We consider the type distribution where the values and the expected opportunity costs are uniformly distributed: $\lambda_{i,a}^{-1} \sim U[0,\ L]$ and $w_{i,a} \sim U[0,\ \lambda_{i,a}^{-1}]$. With $w_{i,a} < \lambda_{i,a}^{-1}$, (A1)-(A3) are satisfied. We set $L = 10$ and $W = 5$, corresponding to scenario where the societal value W is equal to the expected opportunity cost for an average agent to use a resource. We present in the full version of this paper [16] additional results for settings where the societal preference for utilization is weaker or stronger, which show the robustness of the contingent payment mechanisms.

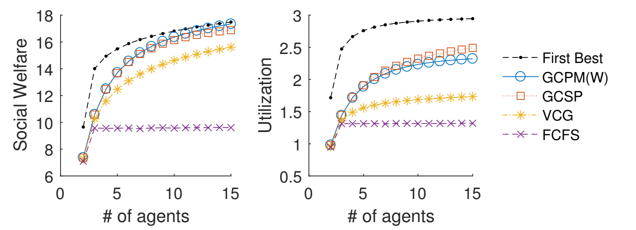
Single Resource Assignment. We first study the assignment of a single resource. Varying the number of agents from 2 to 15, we compute the average welfare and utilization over 10,000 randomly generated profiles under the CP(W), CSP, SP mechanisms, and other benchmarks. See Figure 8. The *First-Best* benchmark is the highest achievable welfare and utilization, subject to the assumptions of IR and ND. The *Random* benchmark assigns the resource at random to one of the agents without charging any payment, modeling the first-come-first-serve reservation systems.

Figure 8a shows that the CP(W) mechanism achieves slightly higher welfare than the CSP mechanism, and is very close to the first-best welfare. Both CP(W) and CSP achieve better social welfare than the SP auction. The average utilization under the mechanisms



(a) Social Welfare **(b) Utilization**

Figure 8: Social welfare and utilization for a single resource.



(a) Social Welfare **(b) Utilization**

Figure 9: Social welfare and utilization for three resources.

is shown in Figure 8b. In comparison to the SP auction, both CSP and CP(W) mechanisms achieve significantly higher utilization. Note that without using any base payment, CSP still achieves significantly higher welfare than random assignment, which can be considered as the status-quo of many real-life reservation systems.

Multiple Heterogeneous Resources. We compare in Figure 9 the social welfare and utilization (expected number of utilized resources) for assigning 3 heterogeneous resources, as the number of agents varies from 2 to 15. The *First Come First Serve* (FCFS) benchmark allows each agent to choose her favorite remaining resource as they arrive in a random order, and does not charge any payments.

7 CONCLUSION

We study the problem of resource assignment where agents have uncertainty about their values and where it is in the interest of the society or the planner that resources be used and not wasted. The CP(W) mechanism optimizes social welfare for assigning a single resource, and can be generalized to assign multiple heterogeneous resources. Simulation results demonstrate the effectiveness and robustness of the contingent payment mechanisms.

Interesting directions for future work include (i) generalizing the model to allow for more than two time periods, where agents may arrive asynchronously, uncertainty unfolds gradually over time, and resources can be re-allocated, (ii) repeated assignments of resources (e.g. weekly spinning classes) using points, and (iii) folding in considerations from behavioral economics, understanding the impact of present-bias on resource utilization, and designing commitment devices through contingent payment mechanisms.

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REFERENCES

[1] Saeed Alaei, Kamal Jain, and Azarakhsh Malekian. 2016. Competitive equilibria in two-sided matching markets with general utility functions. *Operations Research* 64, 3 (2016), 638–645.

[2] Alp E Atakan and Mehmet Ekmekci. 2014. Auctions, actions, and the failure of information aggregation. *American Economic Review* 104, 7 (2014).

[3] Dirk Bergemann and Juuso Välimäki. 2010. The dynamic pivot mechanism. *Econometrica* 78, 2 (2010), 771–789.

[4] Patrick Bolton and Mathias Dewatripont. 2005. *Contract theory.* MIT press.

[5] Ruggiero Cavallo, David C. Parkes, and Satinder Singh. 2010. *Efficient Mechanisms with Dynamic Populations and Dynamic Types.* Technical Report. Harvard University.

[6] Richard E Caves. 2003. Contracts between art and commerce. *Journal of economic Perspectives* (2003), 73–84.

[7] Edward H Clarke. 1971. Multipart pricing of public goods. *Public choice* 11, 1 (1971), 17–33.

[8] Pascal Courty and Hao Li. 2000. Sequential screening. *The Review of Economic Studies* 67, 4 (2000), 697–717.

[9] Rahul Deb and Debasis Mishra. 2014. Implementation with contingent contracts. *Econometrica* 82, 6 (2014), 2371–2393.

[10] Gabrielle Demange and David Gale. 1985. The strategy structure of two-sided matching markets. *Econometrica: Journal of the Econometric Society* (1985), 873–888.

[11] Mehmet Ekmekci, Nenad Kos, and Rakesh Vohra. 2016. Just Enough or All: Selling a Firm. *American Economic Journal: Microeconomics* 8, 3 (2016), 223–56.

[12] Theodore Groves. 1973. Incentives in teams. *Econometrica: Journal of the Econometric Society* (1973), 617–631.

[13] Oliver Hart and Bengt Holmstrom. 1986. *The theory of contracts.* Department of Economics, MIT.

[14] Kenneth Hendricks and Robert H Porter. 1988. An empirical study of an auction with asymmetric information. *The American Economic Review* (1988), 865–883.

[15] Bengt Holmstrom. 1979. Moral Hazard and Observability. *The Bell Journal of Economics* 10, 1 (1979), 74–91.

[16] Hongyao Ma, Reshef Meir, David C. Parkes, and James Zou. 2018. Contingent Payment Mechanisms for Resource Utilization. *arXiv preprint arXiv:1607.06511* (2018).

[17] Hongyao Ma, Valentin Robu, Na Li, and David C. Parkes. 2016. Incentivizing Reliability in Demand-Side Response. In *Proceedings of The 25th International Joint Conference on Artificial Intelligence (IJCAI'16).* 352–358.

[18] Hongyao Ma, Valentin Robu, and David C. Parkes. 2017. Generalizing Demand Response Through Reward Bidding. In *Procedings of the 16th International Conference on Autonomous Agents and Multiagent Systems (AAMAS'17).* 60–68.

[19] Reshef Meir, Hongyao Ma, and Valentin Robu. 2017. Contract Design for Energy Demand Response. In *IJCAI'17.* 1202–1208.

[20] David C. Parkes and Satinder Singh. 2003. An MDP-Based Approach to Online Mechanism Design. In *Proc. 17th Annual Conf. on Neural Information Processing Systems (NIPS'03).*

[21] Ryan Porter, Amir Ronen, Yoav Shoham, and Moshe Tennenholtz. 2008. Fault tolerant mechanism design. *Artificial Intelligence* 172, 15 (2008), 1783–1799.

[22] Andrzej Skrzypacz. 2013. Auctions with contingent payments— an overview. *International Journal of Industrial Organization* 31, 5 (2013), 666–675.

[23] Lars Stole. 2001. Lectures on the Theory of Contracts and Organizations. *Unpublished monograph* (2001).

[24] Hal R Varian. 2007. Position auctions. *international Journal of industrial Organization* 25, 6 (2007), 1163–1178.

[25] William Vickrey. 1961. Counterspeculation, auctions, and competitive sealed tenders. *The Journal of finance* 16, 1 (1961), 8–37.