Groups Versus Coalitions: On the Relative Expressivity of GAL and CAL

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ABSTRACT

Group Announcement Logic (GAL) and Coalition Announcement Logic (CAL) were proposed to study effects of public announcements by groups of agents on knowledge in multiagent systems. Both logics have operators that quantify over such announcements. In GAL, it is possible to express that 'a group of agents G has a (truthful) announcement such that after this announcement, some property A holds'; for example, A may involve some agents in G gaining additional knowledge, while agents outside G remain ignorant. In CAL, the meaning of the coalition announcement operator is subtly different: it says that 'G has an announcement such that, whatever else the agents outside G announce simultaneously, some property A is guaranteed to hold after the joint announcement'. It has been open for some time whether GAL and CAL are equally expressive. We show that this is not the case: there is a property expressible in GAL that is not expressible in CAL. It is still an open question whether CAL is subsumed by GAL, or whether the two logics have incomparable expressive power.

KEYWORDS

Coalition announcement logic; group announcement logic; public announcements; dynamic epistemic logic

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1 INTRODUCTION

Public announcements are dynamic epistemic operators that allow us to reason about the effects of agents simultaneously and publicly acquiring some truthful information. An epistemic logic extended with such a kind of operators is called public announcement logic (PAL) [10, 14]. In [5] the authors consider a variant of PAL with quantification over public announcements. They extend the language of PAL with formulas $\Box\phi$ that can express the following: 'after any public announcement, ϕ holds'. The resulting logic is called arbitrary public announcement logic (APAL). Two other notable logics of quantified announcements are group announcement logic (GAL) [1] and coalition announcement logic (CAL) [2]. In the former the group announcement construct $[\![G]\!]\phi$ means that 'whatever agents from group G announce, ϕ holds afterwards'. In the latter, there is an alternation of quantifiers. Coalition announcement constructs $[\langle G\rangle]\phi$ are read as 'whatever agents from coalition G announce, other agents outside of the coalition (i.e. agents from $A \setminus G$) can simultaneously announce something such that ϕ holds afterwards'. In the case of GAL and CAL, announcements are conjunctions of statements of knowledge of agents from corresponding groups. In other words, agents announce only what they know.

The GAL and CAL modalities have been applied to specify notions and problems in imperfect information games, in security protocols (what can the principals guarantee after any protocol?), in epistemic game theory, and in general in reasoning about games and strategies (coalition announcements formalize *playability*) [1, 2, 9, 11]. There are also relations between GAL and logics of agency ATL/ATEL [4, 12], and between GAL and STIT/Next-STIT [7], where we should observe that to get a GAL-like (or CAL-like) effect we need to combine agency modalities in such logics with temporal modalities.

A topic of interest for announcement logics is their relative *expressivity*. It is known that epistemic logic is equally expressive as public announcement logic [10], and that APAL, GAL, and CAL are more expressive than PAL [1, 5]. However, the relative expressivity of APAL, GAL, and CAL has been an open question for almost a decade [1, 13]. In this paper we determine that CAL is not at least as expressive as GAL. This is one direction needed to determine the relative expressivity of GAL and CAL. The other direction is left for future research. A nice 'side-effect' of our result is that APAL is not at least as expressive as GAL, which was mentioned as an open question in [1]. As is well-known, dynamic epistemic logics have various and surprising expressivity results, due to group epistemic phenomena and to the interaction of dynamic and epistemic features; and those results are often non-trivial to establish [1, 5, 8, 14]. We hope that our work helps towards their further expansion.

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2 PRELIMINARIES

Let P be a countable set of atomic propositions, and let A be a finite set of agents.

Definition 2.1. The combined syntax of *epistemic logic* ($Lang_{EL}$), *public announcement logic* ($Lang_{PAL}$), *arbitrary public announcement logic* ($Lang_{APAL}$), *group announcement logic* ($Lang_{GAL}$) and *coalition announcement logic* ($Lang_{CAL}$) is given by the following recursion:

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \mathrm{K}_i\phi \mid [\phi]\phi \mid \Box\phi \mid [\![G]\!]\phi \mid [\langle G\rangle]\phi,$$

where $p \in P$, $i \in A$, and $G \subseteq A$. The language of epistemic logic consists of the formulas not including $[\alpha]\phi$, $\Box\phi$, $[\![G]\!]\phi$, and $[\langle G\rangle]\phi$. $Lang_{PAL}$ is without $[\![G]\!]\phi$, $\Box\phi$, and $[\langle G\rangle]\phi$. The language of arbitrary public announcement logic consists of formulas not including $[\![G]\!]\phi$ and $[\langle G\rangle]\phi$. $Lang_{GAL}$ is the language without $[\langle G\rangle]\phi$ and $\Box\phi$. $Lang_{CAL}$ consists of formulas not including $[\![G]\!]\phi$ and $\Box\phi$.

We use standard abbreviations $\top, \bot, \vee, \rightarrow, \leftrightarrow$, as well as the dual operators $\mathrm{L}_i\phi = \neg\mathrm{K}_i\neg\phi$, $\langle\alpha\rangle\phi = \neg[\alpha]\neg\phi$, $\Diamond\phi = \neg\Box\neg\phi$, $\langle\!\langle G\rangle\!\rangle\phi = \neg[\![G]\!]\neg\phi$, and $\langle\![G]\!\rangle\phi = \neg[\langle G\rangle]\neg\phi$.

Formulas of all logics considered in the paper are interpreted on epistemic models.

Definition 2.2. An *epistemic model* is a tuple $M = (S, \sim, V)$, where S is a non-empty set of states, for each agent $i \in A$, $\sim_i \subseteq S^2$ is an equivalence relation, and $V : P \longrightarrow 2^S$ is a valuation function. For $s \in S$, a pointed model $M_s = (S, \sim, V, s)$ specifies a state of evaluation. Given $X \subseteq S$, we write $M^X = (X, \sim^X, V^X)$, where for all $i \in A$, $\sim_i^X = \sim_i \cap X^2$, and for all $p \in P$, $V^X(p) = V(p) \cap X$. An i-equivalence class in model M is denoted as $[s]_i^M = \{t \in S \mid t \sim_i s\}$. We denote the elements of the tuple as $M = (S^M, \sim^M, V^M)$.

Definition 2.3. Given a pointed epistemic model, $M_s = (S, \sim, V, s)$, the *semantics* of CAL, GAL, and APAL is defined by induction on formula structure.

$M_s \models p$	$\Leftrightarrow$	$s \in V(p)$
$M_s \models \neg\phi$	$\Leftrightarrow$	$M_s \not\models \phi$
$M_s \models \phi \wedge \psi$	$\Leftrightarrow$	$M_s \models \phi$ and $M_s \models \psi$
$M_s \models \mathrm{K}_i\phi$	$\Leftrightarrow$	$\forall t \in [s]_i,\ M_t \models \phi$
$M_s \models [\alpha]\phi$	$\Leftrightarrow$	$M_s \not\models \alpha$ or $M_s^{\Vert\alpha\Vert_M} \models \phi$
$M_s \models \Box\phi$	$\Leftrightarrow$	$\forall \alpha \in Lang_{EL} : M_s \models [\alpha]\phi$
$M_s \models [\![G]\!]\phi$	$\Leftrightarrow$	$\forall \alpha \in !(G)$, if $M_s \models \alpha$ then $M_s^{\Vert\alpha\Vert_M} \models \phi$
$M_s \models [\langle G\rangle]\phi$	$\Leftrightarrow$	$\forall \alpha \in !(G)$, $\exists \beta \in !(A \backslash G)$ s.t.
		$M_s \models \alpha \Rightarrow M_s \models \beta$ and $M_s^{\Vert\alpha\wedge\beta\Vert_M} \models \phi$

where $\Vert\alpha\Vert_M = \{s \in S^M \mid M_s \models \alpha\}$. The set $!(G)$ consists of formulas of type $\bigwedge_{i\in G}\mathrm{K}_i\phi_i$ where $\phi_i \in Lang_{EL}$.

The proofs in the paper will rely on the notion of bisimulation, so we will repeat the well-known definition and results here [6].

Definition 2.4. Given two pointed models M_s and N_t, we say M_s and N_t are *bisimilar* ($M_s \simeq N_t$) if there exists a relation $\mathcal{B} \subseteq S^M \times S^N$ such that $(s, t) \in \mathcal{B}$ and for all $(u, v) \in \mathcal{B}$:

- for all $p \in P$, $u \in V^M(p)$ if and only if $v \in V^N(p)$;
- for all $i \in A$, for all $u' \in [u]_i^M$ there exists $v' \in [v]_i^N$ where $(u', v') \in \mathcal{B}$;
- for all $i \in A$, for all $v' \in [v]_i^N$ there exists $u' \in [u]_i^M$ where $(u', v') \in \mathcal{B}$;

We refer to the relation $\mathcal{B}$ as a *bisimulation.* n-bisimulation is a relation between states where the mutual simulation can be maintained for n moves.

For $\mathcal{L} = Lang_{PAL}, Lang_{APAL}, Lang_{GAL}, Lang_{CAL}$ we have that $\mathcal{L}$-bisimilar implies $\mathcal{L}$-modal equivalent, and, on finite models, vice versa. M_s and N_t are $\mathcal{L}$-modally equivalent iff for all $\phi \in \mathcal{L}$, $M_s \models \phi$ iff $N_t \models \phi$. n-bisimilar models do not distinguish epistemic formulas of modal depth (nesting of modalities) n.

The focus of the paper is the expressivity result.

Definition 2.5. Given two languages $Lang_1$ and $Lang_2$, we say that $Lang_1$ *is at least as expressive as* $Lang_2$ ($Lang_1 \geq Lang_2$) iff for every $\psi_2 \in Lang_2$ there is an equivalent $\psi_1 \in Lang_1$. We also write $Lang_1 \not\geq Lang_2$ if $Lang_1$ is not at least as expressive as $Lang_2$. If $Lang_1 \not\geq Lang_2$ and $Lang_2 \not\geq Lang_1$, we say that $Lang_1$ and $Lang_2$ are incomparable.

We will abuse notation and write $L_1 \geq L_2$ instead of $Lang_{L_1} \geq Lang_{L_2}$, and $L_1 \not\geq L_2$ instead of $Lang_{L_1} \not\geq Lang_{L_2}$.

3 MOTIVATION

The semantics of group and coalition announcement operators suggests the latter can be defined in terms of the former as $\langle\![G]\!\rangle\phi \leftrightarrow \langle\!\langle G\rangle\!\rangle[\![A \setminus G]\!]\phi$. Validity of this formula was mentioned as an open question in [13] and [3]. We show that the formula is not valid by presenting a counterexample.

There are two main points in the intuition behind the counterexample. First, an announcement by G may make some states bisimilar and thus indistinguishable for $A \setminus G$. In such a way, agents from $A \setminus G$ may 'lose' some strategies they had in the original model. And the second point is that an announcement by a group of agents $A \setminus G$ can influence not only epistemic relations of their opponents but of agents from $A \setminus G$ as well.

Proposition 3.1. *$\langle\!\langle G\rangle\!\rangle[\![A \setminus G]\!]\phi \leftrightarrow \langle\![G]\!\rangle\phi$ is not valid.*

Proof. We present a counterexample (Figures 1, 2, and 3) to the contraposition $[\langle G\rangle]\phi \rightarrow [\![G]\!]\langle\!\langle A \setminus G\rangle\!\rangle\phi$ of one direction of the equivalence, involving three agents a, b, and c, and one atom p. We use black squares for states where p holds, and we use white square for states where $\neg p$ holds. Solid lines are agent a's relations, dashed ones are agent b's, and agent c cannot distinguish states in the same dotted box. Reflexive and transitive connections are omitted.

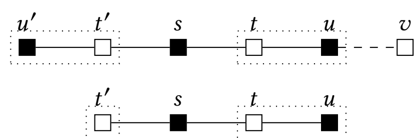

Figure 1: Models M (above) and N (below)

Let $\phi := \mathrm{L}_a\mathrm{K}_c\neg p \wedge \mathrm{L}_a(\mathrm{L}_c p \wedge \mathrm{L}_c\neg p)$. This formula is a distinguishing formula of state s of model N, i.e. ϕ is true only in N_s and nowhere else in this proof. First, we show that $M_s \models [\langle\{a\}\rangle]\varphi$. This means that for every $\alpha \in !(\{a\})$, there are $\beta \in !(\{b\})$ and $\gamma \in !(\{c\})$ such that $M_s \models \alpha \rightarrow \langle\alpha \wedge \beta \wedge \gamma\rangle\varphi$. Agent a can update M_s in two non-equivalent ways: either leaving the whole model as it is, or restricting it to $\{u', t', s, t, u\}$. On the other hand, because the intersection of the relations b and c is the identity relation, agents $\{b, c\}$ can force any possible submodel of M that include s. For either of a's announcements, agents $\{b, c\}$ can make an announcement such that the model is reduced to $\{t', s, t, u\}$. Particularly, b announces a formula true $\{u', t', s, t, u\}$, and c announces a formula true in $\{t', s, t, u, v\}$. Such a joint announcement results in model N, and $N_s \models \varphi$.

Now, let us show that $M_s \not\models [\![\{a\}]\!]\langle\!\langle\{b, c\}\rangle\!\rangle\varphi$, which is equivalent, by the semantics, to $M_s \models \langle\!\langle\{a\}\rangle\!\rangle[\![\{b, c\}]\!]\neg\varphi$. Let a announce $\alpha := \mathrm{K}_a(\neg p \rightarrow \mathrm{L}_c p)$ that is true in $\{u', t', s, t, u\}$. Such an announcement makes states u and u', and t and t' bisimilar. The resulting model and a smaller bisimilar one are presented in Figure 2.

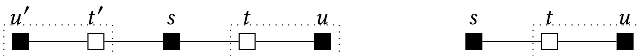

Figure 2: Models $M^{\|\alpha\|_M}$ (left) and M^2 (right)

Thus $M^{\|\alpha\|_M} \simeq M^2$. In model M^2_s coalition $\{b, c\}$ can force the following updated models: $\{s, t, u\}$, $\{s, t\}$, and $\{s\}$. Results of these updates are presented in Figure 3.

Figure 3: Subsequent updates of model M^2_s

It is easy to check that none of the models from Figure 3 satisfy φ. Hence, $M_s \not\models [\![\{a\}]\!]\langle\!\langle\{b, c\}\rangle\!\rangle\varphi$, and, combining these two evaluations, it follows that $[\langle\{a\}\rangle]\varphi \rightarrow [\![\{a\}]\!]\langle\!\langle\{b, c\}\rangle\!\rangle\varphi$ is not valid. ◻

Proposition 3.1 shows that a simple embedding of CAL in GAL is invalid. Of course, this does not rule out that there are other ways to investigate whether GAL is at least as expressive as CAL. From the problem of the relative expressivity of GAL and CAL we will only tackle the other direction: we will show that CAL is not at least as expressive as GAL. On this theme we now continue.

4 FORMULA GAMES

In order to show that CAL is not at least as expressive as GAL (there are properties that can be expressed in GAL but not in CAL) we will define a set of pointed models $\mathcal{M}$ and a formula $\xi \in Lang_{GAL}$ such that for all $\psi \in Lang_{CAL}$, $\mathcal{M}^{\xi} \neq \mathcal{M}^{\psi}$, where $\mathcal{M}^{\phi} = \{M_s \in \mathcal{M} \mid M_s \models \phi\}$. The proof will use a game between two players evaluating a $Lang_{CAL}$ formula on a pointed model. The universal player is trying to show that the formula is false, and the existential player is trying to show that the formula is true.

To describe the game, we must first extend the syntax of $Lang_{CAL}$ to separate out the inherent alternation in the $[\langle G\rangle]$ operator.

Definition 4.1. The *half coalition operators* are formulas of the type $\rrparenthesis G, \alpha \llparenthesis\, \phi$, where

$$M_s \models \rrparenthesis G, \alpha \llparenthesis\, \phi \quad \Leftrightarrow \quad \forall \beta \in !(G): \text{ if } M_s \models \beta, \text{ then } M_s \models \alpha \text{ and } M_s^{\|\alpha \wedge \beta\|_M} \models \phi.$$

We use the abbreviation $\llparenthesis\, G, \alpha \rrparenthesis \phi$ for $\neg \rrparenthesis G, \alpha \llparenthesis\, \neg\phi$.

The half coalition operators separate the coalition's announcement from the anti-coalition's response, so we can describe them using separate moves in a game.

Definition 4.2. The syntax of *negation normal form* (NNF) is:

$$\phi ::= \begin{array}{l} \top \mid \phi \wedge \phi \mid \mathrm{K}_i\phi \mid \mid [\![G]\!]\phi \mid [\langle G\rangle]\phi \mid \rrparenthesis G, \phi \llparenthesis\, \phi \\ \mid \quad \bot \mid p \mid \neg p \mid \phi \vee \phi \mid \mathrm{L}_i\phi \mid [\phi]\phi \mid \langle\!\langle G\rangle\!\rangle\phi \mid \llparenthesis G \rrparenthesis\phi \mid \llparenthesis\, G, \phi \rrparenthesis\phi \end{array}$$

We consider two kinds of negation normal form. In *universal negation normal form* (UNNF) the modalities are 'necessity'-type, i.e., in the first row of the BNF grammar above, whereas it is *existential negation normal form* (ENNF) if the modalities are 'diamond'-type, i.e., in the second row of the BNF grammar above.

Lemma 4.3. *Every formula of $Lang_{GAL} \cup Lang_{CAL}$ is equivalent to a formula in negation normal form.*

Proof. The proof is straightforward and involves moving negations inside all non-atomic operators via the translation τ:

$$\begin{array}{lcrlcr}
(\neg(\phi \wedge \psi))^{\tau} & = & (\neg\phi)^{\tau} \vee (\neg\psi)^{\tau} & (\phi \wedge \psi)^{\tau} & = & \phi^{\tau} \wedge \psi^{\tau} \\
(\neg \mathrm{K}_i\phi)^{\tau} & = & \mathrm{L}_i(\neg\phi)^{\tau} & (\mathrm{K}_i\phi)^{\tau} & = & \mathrm{K}_i\phi^{\tau} \\
(\neg[\alpha]\phi)^{\tau} & = & \alpha^{\tau} \wedge ([\alpha]\neg\phi)^{\tau} & ([\alpha]\phi)^{\tau} & = & [\alpha^{\tau}]\phi^{\tau} \\
(\neg[\![G]\!]\phi)^{\tau} & = & \langle\!\langle G\rangle\!\rangle(\neg\phi)^{\tau} & ([\![G]\!]\phi)^{\tau} & = & [\![G]\!]\phi^{\tau} \\
(\neg[\langle G\rangle]\phi)^{\tau} & = & \llparenthesis G \rrparenthesis(\neg\phi)^{\tau} & ([\langle G\rangle]\phi)^{\tau} & = & [\langle G\rangle]\phi^{\tau} \\
(\neg\neg\phi)^{\tau} & = & \phi^{\tau} & & &
\end{array}$$

We note that the operators $\llparenthesis\, G, \phi \rrparenthesis$, $\rrparenthesis G, \phi \llparenthesis$, $\top$ and $\bot$ will not appear in the image of this translation. However they are required as intermediate states in the game structure below. ◻

Now we are ready to define formula games.

Definition 4.4. Let $M_s = (S, \sim, V, s)$ be a pointed model, and suppose that $\mathcal{M}$ is the set of pointed submodels of M_s, M_t^X, where $X \subseteq S$ and $t \in X$. Given a formula ϕ in negation normal form, a *formula game for ϕ* over M_s is a tuple $\mathcal{G}^{\phi}_{M_s} = (V_{\forall}, V_{\exists}, E, v)$ where:

- $V_{\forall} = \{(N_t, \psi) \mid N_t \in \mathcal{M},\ \psi \in \text{UNNF}\} \cup \{(N_t, X, \alpha, \psi) \mid N_t \in \mathcal{M},\ X \subseteq S^N,\ \alpha, \psi \in \text{NNF}\}$ is a set of vertices of player $\forall$.
- $V_{\exists} = \{(N_t, \psi) \mid N_t \in \mathcal{M},\ \psi \in \text{ENNF}\}$ is a set of vertices belonging to player $\exists$.
- $E \subset (V_{\forall} \cup V_{\exists})^2$ is the set of edges, where:

$$E = \bigcup \left[\begin{array}{l}
\{((N_t, p), (N_t, \top)), ((N_t, \neg q), (N_t, \top)) \\
\qquad \mid t \in V^M(p) \text{ and } t \notin V^M(q)\} \\
\{((N_t, p), (N_t, \bot)), ((N_t, \neg q), (N_t, \bot)) \\
\qquad \mid t \notin V^M(p) \text{ and } t \in V^M(q)\} \\
\{((N_t, \psi \wedge \chi), (N_t, \psi)),\ ((N_t, \psi \wedge \chi), (N_t, \chi)) \mid N_t \in \mathcal{M}\} \\
\{((N_t, \psi \vee \chi), (N_t, \psi)),\ ((N_t, \psi \vee \chi), (N_t, \chi)) \mid N_t \in \mathcal{M}\} \\
\{((N_t, \mathrm{K}_i\psi), (N_u, \psi)) \mid N_t \in \mathcal{M} \text{ and } t \sim_i^N u\} \\
\{((N_t, \mathrm{L}_i\psi), (N_u, \psi)) \mid N_t \in \mathcal{M} \text{ and } t \sim_i^N u\} \\
\{((N_t, [\alpha]\psi), (N_t, X, \alpha, \psi)) \mid N_t \in \mathcal{M},\ X \subseteq S^N\} \\
\{((N_t, X, \alpha, \psi), (N_x, \alpha)) \mid x \in X\} \\
\{((N_t, X, \alpha, \psi), (N_x, (\neg\alpha)^{\tau})) \mid x \in S^N \backslash X\} \\
\{((N_t, X, \alpha, \psi), (N_t^X, \psi)) \mid t \in X\} \\
\{((N_t, [\![G]\!]\psi), (N_t, [\alpha]\psi)) \mid N_t \in \mathcal{M},\ \alpha \in !(G)\} \\
\{((N_t, \langle\!\langle G\rangle\!\rangle\psi), (N_t, [\alpha]\psi)) \mid N_t \in \mathcal{M},\ \alpha \in !(G)\} \\
\{((N_t, [\langle G\rangle]\psi), (N_t, \llparenthesis\, A \setminus G, \alpha \rrparenthesis\psi)) \mid N_t \in \mathcal{M},\ \alpha \in !(G)\} \\
\{((N_t, \llparenthesis G \rrparenthesis\psi), (N_t, \rrparenthesis A \setminus G, \alpha \llparenthesis\, \psi)) \mid N_t \in \mathcal{M},\ \alpha \in !(G)\} \\
\{((N_t, \rrparenthesis G, \alpha \llparenthesis\, \psi), (N_t, [\alpha \wedge \beta]\psi)) \mid N_t \in \mathcal{M},\ \beta \in !(A \backslash G)\} \\
\{((N_t, \llparenthesis\, G, \alpha \rrparenthesis\psi), (N_t, [\alpha \wedge \beta]\psi)) \mid N_t \in \mathcal{M},\ \beta \in !(A \backslash G)\}
\end{array}\right]$$

- The initial vertex is $v = (M_s, \phi)$.

The game is played between two players $\forall$ and $\exists$, and a play consists of a sequence of states, $\overline{v} = v_0, \ldots, v_n$, where $v_0 = v$. The sequence is built by the players, so that $(v_m, v_{m+1}) \in E$ and if $v_m \in V_\forall$, player $\forall$ chooses v_{m+1} and if $v_m \in V_\exists$, player $\exists$ chooses v_{m+1}. A player wins the game if their opponent is unable to move (so the $\forall$-player is trying to force the game to a $\bot$-state, and the $\exists$-player is trying to force the game to a $\top$-state).

Lemma 4.5. (1) *Every play of the game $\mathcal{G}^{\phi}_{M_s}$ is finite.*
(2) *Player $\exists$ has a winning strategy in the game $\mathcal{G}^{\phi}_{M_s}$ if and only if $M_s \models \phi$.*

Proof. (1) To show every play is finite we focus on the second element of the tuple. We note that all formulas can be ordered by the number of second-order operators ($[\![G]\!]$, or $[\langle G\rangle]$) they contain first, and then by the size of formulas. It can be shown that this ordering is well-founded, and the play can only descend according to this order.

(2) ($\Longleftarrow$) If $M_s \models \phi$, then player $\exists$ can always choose a strategy that preserves the invariant $v_i = (N_t, \psi)$ implies $N_t \models \psi$. The only move that requires special attention is when $\psi = [\alpha]\chi$. Here, player $\exists$ must chose a vertex where $X = \|\alpha\|_N$. If this holds for every state chosen by player $\exists$, we can see that player $\forall$ can't help but preserve the invariant. As the game is finite, it must eventually end in a vertex $(N_t, \top)$ so player $\exists$ wins.
($\Longrightarrow$) Suppose, for contrapositive, that $M_s \not\models \phi$. Then as above, player $\forall$ can always choose vertices that preserve the invariant $v_i = (N_t, \psi)$ implies $N_t \not\models \psi$, and player $\exists$ cannot break the invariant. As above, for $[\alpha]\chi$ vertices, $\exists$ must choose a vertex where $X = \|\alpha\|_N$. As for any other move, $\forall$ will be able to some (N_t, ψ) where $N_t \not\models \psi$, preserving the invariant. Therefore the game will end at some vertex $(N_t, \bot)$ and player $\forall$ wins. □

The proof will now proceed by constructing a set of pointed models, $\mathcal{M}$, that is divided into $\mathcal{M}_1$ and $\mathcal{M}_2$ according to some property expressible in the language $Lang_{GAL}$. We then suppose for contradiction that there is a formula, ϕ, in $Lang_{CAL}$ that can express this property. Therefore, it should be the case that in the formula game $\mathcal{G}^{\phi}_{M_s}$, $\exists$ has a winning strategy if $M_s \in \mathcal{M}_1$, and $\forall$ has a winning strategy if $M_s \in \mathcal{M}_2$. We have shown that we can play these winning strategies against one another in such a way that there is some model M_s where both $\forall$ and $\exists$ have a winning strategy, giving the required contradiction.

5 CAL $\not\geq$ GAL

We will use three-agent models (for single agent models all languages are trivially equivalent to epistemic logic, and we have not found an effective two-agent encoding), and refer to the agents as a, b and c, so $A = \{a, b, c\}$. We proceed by first identifying a set of interesting models, and specifying a semantic property over these models which is expressible in $Lang_{GAL}$. After that, we show that any attempt to express this property in $Lang_{CAL}$ is doomed to failure.

5.1 Chain Models

We are interested in a special class of epistemic models.

Definition 5.1. A *chain* (l, u) is an epistemic model $M = (S, \sim, V)$ where:

- $S = \{l, l+1, \ldots, u-1, u\} \subset \mathbb{Z}$ is a finite set of consecutive integers.
- $x \sim_a y$ and $y \sim_a x$ if and only if $y = x + 1$ and x is even.
- $x \sim_b y$ and $y \sim_b x$ if and only if $y = x + 1$ and x is odd.
- $z - 1 \sim_c z \sim_c z + 1$ if and only if $z \bmod 3 = 1$ (and symmetric closure of that).
- $V(p) = \{3k, 3k - 1 \in S \mid k \in \mathbb{Z}\}$.

For example, chain $(1, 12)$ is depicted in Figure 4.

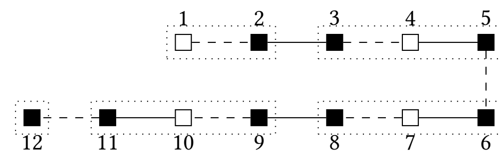

Figure 4: A representation of a chain. The atom p is true at the black states, and false at the white states. Agent a's accessibility relation is a solid line, agent b's accessibility relation is a dashed line, and agent c cannot distinguish states in the same dotted box.

Clearly, chains are very regular and differ only in their extremities. We can give a classification of chain models that is based on the types of rightmost and leftmost states.

Let some model (x, y) be given.

- If $x \bmod 6 = 1$ (resp. $y \bmod 6 = 4$), then $\mathrm{K}_a\neg p$ is true at the left (right) most state.
- If $x \bmod 6 = 2$, (resp. $y \bmod 6 = 3$), then $\mathrm{K}_b\mathrm{K}_a p$ is true at the left (right) most state.
- If $x \bmod 6 = 3$ (resp. $y \bmod 6 = 2$), then $\mathrm{K}_a p$ is true at the left (right) most state. Note that for such an x (y), chain (x, y) is bisimilar to model $(2x - y - 1, y)$ via the bisimulation $\{(x+k, x-k-1) \mid 0 \leq k \leq y-x\}$ (resp. the model $(x, 2y-x+1)$ via the bisimulation $\{(y + k, y - k - 1) \mid 0 \leq k \leq y - x\}$). See Figure 5 for an example. Hence, there is no epistemic formula that can distinguish state x (y) from non-extreme states in larger chains.
- If $x \bmod 6 = 4$ (resp. $y \bmod 6 = 1$), then $\mathrm{K}_b\neg p$ is true at the left (right) most state.
- If $x \bmod 6 = 5$, (resp. $y \bmod 6 = 0$), then $\mathrm{K}_a\mathrm{K}_b p$ is true at the left (right) most state.
- If $x \bmod 6 = 0$, (resp. $y \bmod 6 = 5$), then $\mathrm{K}_b p$ is true at the left (right) most state. As in the case $x \bmod 6 = 3$ ($y \bmod 6 = 2$), model (x, y) is bisimilar to the model $(2x - (y + 1), y)$ (resp. $(x, 2y - x + 1)$) as above.

Therefore, we can describe the *type* of a model (x, y) as the pair $[x \bmod 6, y \bmod 6]$.

In this paper we are primarily interested in models of types $[1, 2]$, $[0, 4]$, and $[0, 2]$ (Figures 6, 7 and 8). We also note that all models with 'unbroken' c-relations are bisimilar to the chain of type $[0, 2]$ (see Figure 8).

In $[1, 2]$- and $[0, 4]$-models we call the state where $\Omega := \mathrm{K}_a\neg p$ holds the *terminal* state. In Figures 6 and 7 the terminal state is the

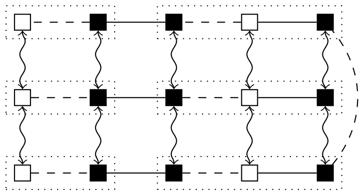

Figure 5: A bisimulation between the chains $(1,5)$ (in the middle) and $(1,10)$ (starting at the top row and wrapping around on the right to the bottom row)

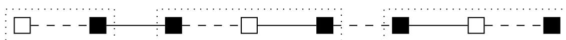

Figure 6: A $[1,2]$-model

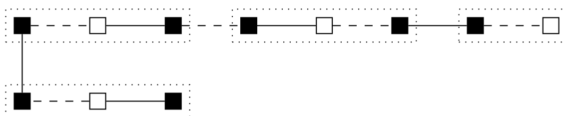

Figure 7: A $[0,4]$-model

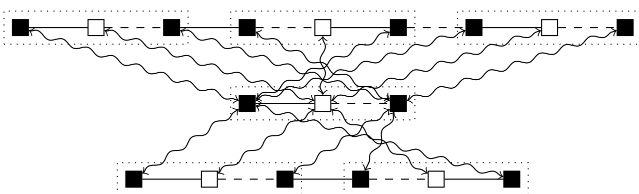

Figure 8: The only chain of type $[0,2]$ up to bisimulation. The top and bottom chains are bisimilar to the middle one

leftmost and rightmost white states respectively. Note that in such models there is only one terminal state.

We use terminal states to define a property expressible in GAL in the next section. Moreover, in models with terminal states it is possible for agents to make group announcements in order to cut chains. For example, to specify a state that is exactly three steps from the Ω state we can use the formula:

$$\Omega + 3 := \mathrm{L}_b\mathrm{L}_a\mathrm{L}_b\Omega \wedge \mathrm{K}_a\mathrm{K}_b\mathrm{K}_a\neg\Omega.$$

See Figure 9 for representation of the formula.

If agent b announces, for example, $\mathrm{K}_b\neg(\Omega+3)$, the updated model will be the one without the b-link with the $\Omega+3$-state. The same kind of an announcement, $\mathrm{K}_a\neg(\Omega+3)$, can be made by agent a, and the updated model will be the one without the a-link with the $\Omega+3$-state. Group $\{a,b\}$ can remove exactly the $\Omega+3$-state by announcing $\mathrm{K}_b\neg(\Omega+3)\wedge\mathrm{K}_a\neg(\Omega+3)$.

Now let us consider non-terminal rightmost and leftmost states in $[1,2]$- and $[1,4]$-chains. They are presented in Figure 10. While presenting the classification of chains, we pointed out that no epistemic formula of size less n can distinguish these states from n-bisimilar ones in larger models. In other words, in order to specify

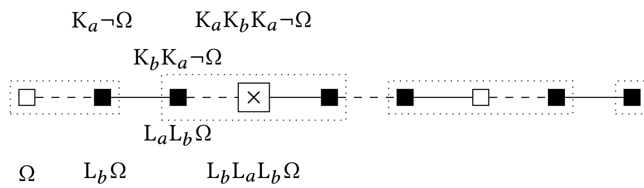

Figure 9: Removing states from a model using Ω

such states, we should refer to the terminal state. Epistemic formulas, however, have a finite size, and hence are true only in chains of some depth, so that we can always find a larger chain of the same type such that any given epistemic formula that was true in the smaller model will be false in the greater one.

Therefore, we use formulas of $Lang_{GAL}$ to describe those non-terminal states, and we call these formulas Mid_a and Mid_b. The former is

$$Mid_a := \mathrm{K}_a p \wedge [\![A]\!](\mathrm{L}_b\neg p \rightarrow \mathrm{K}_a\mathrm{L}_b\neg p),$$

and it holds in the rightmost states of $[1,2]$-models. The latter is

$$Mid_b := \mathrm{K}_b p \wedge [\![A]\!](\mathrm{L}_a\neg p \rightarrow \mathrm{K}_b\mathrm{L}_a\neg p),$$

and it holds in the leftmost states of $[0,4]$-models.

Figure 10: Mid_a and Mid_b

5.2 The Definition of the Property

We start this section with GAL formulas that are valid on a certain class of chain models. First,

$$T(0,2) := \mathrm{K}_a\mathrm{K}_b(\neg p \rightarrow [\![A]\!]((\mathrm{L}_a p \wedge \mathrm{L}_b p) \rightarrow \mathrm{K}_a\mathrm{K}_b\neg(\mathrm{K}_a p \wedge \mathrm{K}_b p)))$$

is true in every state of any $[0,2]$-model (so it is valid on such a model) and false in every state of any other type of chain model. It is therefore distinguishing between types of chain models used in our proof.

Formulas for $[1,2]$- and $[0,4]$-models are as follows:

$$T(1,2) := \neg T(0,2) \wedge [\mathrm{K}_b\neg\Omega_1]T(0,2) \wedge [\neg Mid_b \wedge \mathrm{K}_b\neg\Omega_1]T(0,2),$$

$$T(0,4) := \neg T(0,2) \wedge [\mathrm{K}_b\neg\Omega_1]T(0,2) \wedge [\neg Mid_a \wedge \mathrm{K}_b\neg\Omega_1]T(0,2).$$

Informally, formula $T(1,2)$ means that the current $[1,2]$-model is not bisimilar to the $[0,2]$ one (conjunct $\neg T(0,2)$), and that removing the bit with the terminal state ($[\mathrm{K}_b\neg\Omega_1]T(0,2)$) makes the model bismilar to the $[0,2]$ one (only 'unbroken' c-equivalence classes remain). These two first conjuncts of $T(1,2)$ are the same as those of $T(0,4)$, and the formulas differ only in the last bit. All states in a $[1,2]$-model satisfy $\neg Mid_b$. This is not the case, however, for one of the extreme states of $T(0,4)$. Vice versa for $\neg Mid_a$.

Soundness of these formulas is straightforward to show. We note that the only quantified modalities used are the group announcement operators that appear in $T(0,2)$, Mid_a and Mid_b. Furthermore, as these operators quantify over the entire group of agents, all of these formulas are expressible in $Lang_{GAL}$. Finally we note that none of these formulas mentions agent c.

The actual property we are interested in applies to pointed models. Given a pointed model (l, u) of type $[1, 2]$ with the actual state s, is the terminal node in the a direction from s (s is an $a : \Omega$ state), or the b direction (s is a $b : \Omega$ state)? See Figure 11. We claim that given a model with a specified point and a terminal state, $Lang_{GAL}$ can express whether the pointed state is an $a : \Omega$ state or a $b : \Omega$ state.

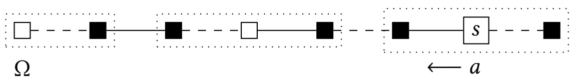

Figure 11: Model $(1, 9)$ with $s = 7$, and s is an $a : \Omega$ state.

The formula that expresses the property of s being an $a : \Omega$ state in a $[1, 2]$-model is:

$$a : \Omega := \bigwedge \begin{bmatrix} \mathrm{K}_b p \rightarrow \langle\!\langle \{c\} \rangle\!\rangle (Mid_b \wedge T(0,4)) \\ \neg p \rightarrow \mathrm{K}_b (p \rightarrow \langle\!\langle \{c\} \rangle\!\rangle (Mid_a \wedge T(1,2))) \\ \mathrm{K}_a p \rightarrow [\![\{c\}]\!] (Mid_a \rightarrow \neg T(1,2)) \end{bmatrix}.$$

Formula $b : \Omega$ can be obtained by swapping subscripts b and a, and formulas $T(0, 4)$ and $T(1, 2)$ in $a : \Omega$.

Theorem 5.2. *Let sets $\mathcal{M}_A$ and $\mathcal{M}_B$ of all $a : \Omega$ and $b : \Omega$ pointed $[1, 2]$-chains be given. Then $M_s \models a : \Omega$ for all $M_s \in \mathcal{M}_A$, and $M_t \not\models a : \Omega$ for all $M_t \in \mathcal{M}_B$.*

Proof. The reader is encouraged to use figures from the previous subsections for reference. Let $M_s \models a : \Omega$ for some $[1, 2]$-chain M_s. Since no conjunction of any two formulas $\mathrm{K}_b p$, $\neg p$, or $\mathrm{K}_a p$ can be true in a pointed chain, we have that either $M_s \models \mathrm{K}_b p$, or $M_s \models \neg p$, or $M_s \models \mathrm{K}_a p$.

Case $\mathrm{K}_b p$. Let $M_s \models \mathrm{K}_b p$. By the construction of chain models, this means that b cannot distinguish two p-states in two adjacent c-equivalence classes, and a considers $\neg p$ possible in the current c-equivalence class. Hence, c can cut b's relation making the current state a Mid_b state. Note that the terminal state remains intact, and thus we have that $T(0, 4)$ holds in the updated model. This means that $M_s \models \langle\!\langle \{c\} \rangle\!\rangle (Mid_b \wedge T(0, 4))$.

Assume that $N_t \models \mathrm{K}_b p$. As N_t is a $b : \Omega$-model, every cut by c either cuts the b-relation, and hence cuts the path to the terminal state, or does not satisfy Mid_b (c cannot make the current state to be extreme). Therefore, $N_t \models [\![\{c\}]\!] (\neg Mid_b \vee \neg T(0, 4))$.

Case $\neg p$. Let $M_s \models \neg p$ and $M_r \models p$ for some r such that $s \sim_b r$. This means that a cannot distinguish two p-states in two adjacent c-equivalence classes, and b considers $\neg p$ possible in the current c-equivalence class. Hence, c can cut a's relation making the current state r a Mid_a state. Note that the terminal state remains intact, and thus we have that $T(1, 2)$ holds in the updated model. This means that $M_r \models p \wedge \langle\!\langle \{c\} \rangle\!\rangle (Mid_a \wedge T(1, 2))$ for some $s \sim_b r$. We can make the latter formula less strict so that it holds in $\neg p$ states as well: $M_r \models p \rightarrow \langle\!\langle \{c\} \rangle\!\rangle (Mid_a \wedge T(1, 2))$. By the construction of chains, there are only two states in b-relation with the current one: a p-state and a $\neg p$-state. Thus, $M_s \models \mathrm{K}_b (p \rightarrow \langle\!\langle \{c\} \rangle\!\rangle (Mid_a \wedge T(1, 2)))$, and we finally have that $M_s \models \neg p \rightarrow \mathrm{K}_b (p \rightarrow \langle\!\langle \{c\} \rangle\!\rangle (Mid_a \wedge T(1, 2)))$.

Assume that $N_t \models \neg p$ and $N_u \models p$ for some u such that $t \sim_b u$. As N_t is a $b : \Omega$-model, N_u is an $a : \Omega$-model. So, every cut by c either cuts the a-relation, and hence cuts the path to the terminal state, or does not satisfy Mid_a (c cannot make the current state to be extreme). Therefore, $N_u \models [\![\{c\}]\!] (\neg Mid_a \vee \neg T(1, 2))$.

Case $\mathrm{K}_a p$. Let $M_s \models \mathrm{K}_a p$. This means that a cannot distinguish two p-states in two c-equivalence classes, and b considers $\neg p$ possible in the current c-equivalence class. Hence, if c cuts a-relation making Mid_a true, she also makes the terminal state inaccessible from the current one. On the other hand, if the terminal state is still accessible from the current state, then in this case the current state does not satisfy Mid_a. This means that $M_s \models [\![\{c\}]\!] (Mid_a \rightarrow \neg T(1, 2))$.

Assume that $N_t \models \mathrm{K}_a p$. As N_t is a $b : \Omega$-model, c has a cut such that Mid_a and $T(1, 2)$ holds. Such a cut 'removes' all c-equivalence classes to the right of the current state, and makes the current state the rightmost state in the updated model. Therefore, $N_t \models \neg [\![\{c\}]\!] (\neg Mid_a \vee \neg T(1, 2))$. □

5.3 The Proof of CAL $\not\geq$ GAL

In this section we show that no CAL formula can capture the property of a pointed model 'being in the $a : \Omega$ state.'

An intuition behind the proof is that CAL operators require all agents announce their knowledge formulas simultaneously. For our chain models, intersection of agents' relations is an identity, and hence if it is possible to force some configuration of an $a : \Omega$-model, then agents together, whether in the same coalition, or divided, can replicate the same configuration in a $b : \Omega$-model. Contrast this to formula $a : \Omega$ in the previous section. The only agent that makes any announcements is c, and her relation is not discerning enough to force isomorphic submodels of $a : \Omega$- and $b : \Omega$-models. If c preserves the terminal state in one class of models, she cannot replicate that announcement in the other class such that the resulting updated models are isomorphic (c cannot cut her own equivalence class to make Ω true in the opposite direction).

Theorem 5.3. *Let sets $\mathcal{M}_A$ and $\mathcal{M}_B$ of all $a : \Omega$ and $b : \Omega$ pointed $[1, 2]$-chains be given. Then for all $\Psi \in Lang_{CAL}$, if $M_s \models \Psi$ for all $M_s \in \mathcal{M}_A$, then there exists some $N_t \in \mathcal{M}_B$ such that $N_t \models \Psi$.*

Proof. We assume, contrary to our claim, that there is a formula Ψ in $Lang_{CAL}$ that is true in all $M_s \in \mathcal{M}_A$ and false in all $N_t \in \mathcal{M}_B$. Let $|\Psi| = n$. We play simultaneous formula games over sufficiently large 2^n-bisimilar models in $\mathcal{M}_A$ and $\mathcal{M}_B$ (to ensure that no epistemic formula can distinguish any two models). We divide the sets of games into $\mathcal{G}_A$ and $\mathcal{G}_B$, where player $\exists$ has a winning strategy for games in $\mathcal{G}_A$, and player $\forall$ has a winning strategy in $\mathcal{G}_B$.

For all moves, except the second order moves, we proceed as follows. If it is an $\exists$-player move, we play the move for $\exists$'s winning strategy on all models in $\mathcal{M}_A$. We also play the corresponding move over $\mathcal{M}_B$. The corresponding move is defined as follows. In the case of disjunction, we choose the same disjunct, and in the case of L move in k-bisimilar states, we consider moves equivalent if the chosen states are are $k - 1$ bisimilar. If it is a $\forall$-player move, we play the move that agrees with the $\forall$ winning strategy in $\mathcal{M}_B$ games, and copy this move in the $\mathcal{M}_A$ games. Thus, we are playing two winning strategies against one another, so the only way the game can end is if one of the sets of games becomes empty (either $\exists$-player or $\forall$-player cannot move). This, however, cannot happen with our chosen property: the formula is identical for both games, and the models are k-bisimilar. Thus if the $\exists$-player cannot move in one game, then she cannot move in the other game contradicting the existence of a winning strategy.

In what follows, we show that we can maintain the following *invariant*: after step i of the formula game, there are infinitely many pairs of models of the same type from $\mathcal{M}_A$ and $\mathcal{M}_B$ that are still 2^{n-i}-bisimilar. In the final step of the game, we end up with some propositional variable on which both classes of models agree. Hence, we have a contradiction since both players have a winning strategy by the assumption.

Cases of boolean and epistemic formulas are trivial.

Case $[\alpha]\psi$: Assume that for some $M_s \in \mathcal{M}_a$, $(M_s, [\alpha]\psi)$ is a winning position for the $\exists$-player. This means that there is a subset X such that (M_s, X, α, ψ) is also a winning position. Let $||\alpha||_N = Y$. We consider two cases. First, if M_s^X is 2^{n-1} bisimilar to N_t^Y, then $\exists$ can play the corresponding move (N_t, Y, α, ψ) in $\mathcal{G}_B$, and the invariant will continue to hold. In the second case, if M_s^X and N_t^Y are not 2^{n-1} bisimilar, there is some $s' \in S^M$ and and some $t' \in S^N$ such that $M_{s'}$ and $N_{t'}$ disagree on the interpretation of α, and s' and t' are within the same 2^{n-1} steps from s and t respectively. W.l.o.g, suppose $M_{s'} \models \alpha$. In this case $\exists$ must still play the corresponding move (N_t, Y, α, ψ) in $\mathcal{G}_B$ (as any alternative to Y would allow $\forall$ to have a winning strategy). However, now player $\forall$ can respond with the moves $(M_{s'}, \alpha)$ in $\mathcal{G}_A$ and $(N_{t'}, (\neg\alpha)^\tau)$ in $\mathcal{G}_B$. By the correctness of the game construction it follows that $\exists$ has a winning strategy in $(N_{t'}, (\neg\alpha)\tau)$, and thus by the determinacy of finite games, $\forall$ has a winning strategy in $(N_{t'}, \alpha)$. Therefore in this case we "jump" the games to the states $(M_{s'}, \alpha)$ and $(N_{t'}, \alpha)$ respectively. Now as s' and t' were the same steps from s and t it follows that $M_{s'}$ is an A model if and only if N_t' is a B-model. Furthermore, since M_s and N_t are 2^n bisimilar, and s' and t' are within 2^{n-1} steps, it follows that $M_{s'}$ and $N_{t'}$ are 2^{n-1} bisimilar, so the invariant continues to hold and the proof proceeds.

Therefore, the game must reach a $[\langle G\rangle]$ operator or a $\langle\!\langle G\rangle\!\rangle$ operator. Note that at this point games may not to be over [1, 2]-models since prior public announcements may have cut chains in various ways. However, this does not affect the proof as we are interested in agents' announcements rather than in chain types.

We will consider only $\langle\!\langle G\rangle\!\rangle$, and the corresponding results for $[\langle G\rangle]$ can be obtained by swapping A to B, and $\exists$ to $\forall$.

Case $\langle\!\langle \{a, b, c\}\rangle\!\rangle\psi$: Assume that for some pointed model $M_s \in \mathcal{M}_A$ there is a half coalition announcement α by agents a, b, and c such that $(M_s, \rangle\!\rangle\emptyset, \alpha\langle\!\langle\, \psi)$ is a winning position for the $\exists$-player. For this node there is only one possible $\forall$-step $(M_s, [\alpha]\psi)$. Since $\mathcal{M}_B$ is infinite, there is a model N_t and an announcement β by a, b, and c, such that M_s^α is isomorphic to N_t^β (see Figure 12 for an example).

Indeed, consider set $S^{\|\alpha\|_M}$. We can enumerate states in the set from left to right. Next, let N^0 be a model such that $S^{N^0} = S^{\|\alpha\|_M}$, $s_{n+1-i} \in V^{N^0}(p)$ iff $s_i \in V^M(p)$, and $s_{n+1-i} \sim_a^{N^0} s_{n-i}$ if and only if $s_i \sim_a^M s_{i+1}$ for all $a \in A$. In other words, we flip model M_s^α from left to right. Note that agents' relations are also flipped: if state s was an a-state, it would become a b-state. Moreover, we can always find a b-model N_t that has N^0 as a submodel. Since agents can together enforce any configuration of N_t, they have a joint announcement β such that N_t^β is isomorphic to M_s^α, where $N_t^\beta = N^0$.

Thus $(N_t, \rangle\!\rangle\emptyset, \beta\langle\!\langle]\psi)$ (and hence $(N_t, [\beta]\psi)$) is a winning position for the $\exists$-player, and she has a winning strategy for a model from $\mathcal{M}_B$. Hence, a contradiction. Note that since agents a, b, and c can together enforce any configuration of a model (up to bisimulation), the argument holds for the case of arbitrary public announcements.

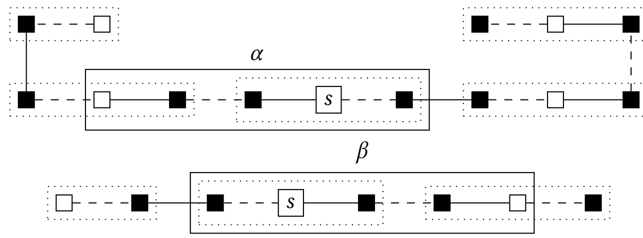

Figure 12: After the announcement of α in an $a : \Omega$-model (above) and β in a $b : \Omega$-model (below), the resulting models are isomorphic

Case $\langle\!\langle \{a, b\}\rangle\!\rangle\psi$: Assume that for some model $M_s \in \mathcal{M}_A$ there is a half coalition announcement α by a and b such that $(M_s, \rangle\!\rangle\{c\}, \alpha\langle\!\langle\, \psi)$ is a winning position for the $\exists$-player. This means that whichever announcement γ by agent c the $\forall$-player chooses, the $\exists$-player is still in a winning position $(M_s, [\alpha\wedge\gamma]\psi)$. There is a model $N_t \in \mathcal{M}_B$ such that for some announcement β by agents a and b it holds that M_s^α is isomorphic to N_t^β, and c has an isomorphic set of possible counter-announcements (see Figure 12). As in the previous case, a and b can together force any configuration of a model. Hence $(N_t, \rangle\!\rangle\{c\}, \beta\langle\!\langle\, \psi)$ is also a winning position for the existential player, and this leads to a contradiction.

Case $\langle\!\langle \{a, c\}\rangle\!\rangle\psi$: Assume that for some $M_s \in \mathcal{M}_A$ there is a half coalition announcement α by a and c such that $(M_s, \rangle\!\rangle\{b\}, \alpha\langle\!\langle\, \psi)$ is a winning position for the $\exists$-player. This implies that $(M_s, [\alpha\wedge\gamma]\psi)$ is also a winning position for the $\exists$-player for any choice of γ by the $\forall$- player. If there is a model $N_t \in \mathcal{M}_B$ such that for some announcement β by a and c it holds that M_s^α is isomorphic to N_t^β, then the reasoning from the previous case applies (see Figure 13, where relevant announcements by b are shown in dashed rectangles).

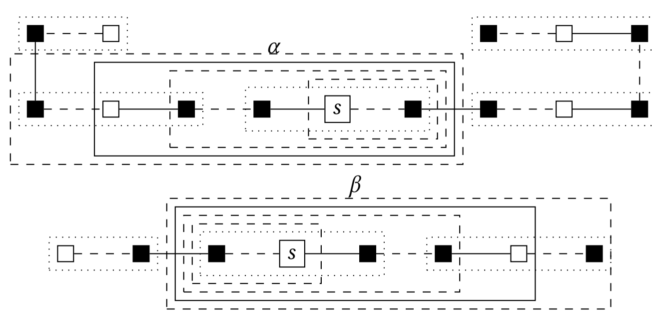

Figure 13: b has the same set of counter-announcements (dashed rectangles) in an $a : \Omega$-model (above) and a $b : \Omega$-model (below)

However, note that $\{a, c\}$ sometimes cannot make such an announcement, because the coalition cannot cut a's relations that are within c-equivalence classes, and α may contain some extreme state. In other words, these a's relations that a and c cannot cut, may have been cut by a previous public announcement (and hence the corresponding state is the rightmost or the leftmost one). Since our chosen $a : \Omega$-model is large enough even after being trimmed by public announcements (i.e. because the invariant holds), there is an a-relation in M_s^α between two c-equivalence classes that b may cut (due to the universal quantification of b's counter-announcements). Moreover, a submodel $M_s^{\alpha'}$ of M_s^α that is restricted by that b-cut, can also be forced by $\{a, c\}$ (because c can cut this relation as well). Thus, replicating the corresponding move and the b-cut in N_t via an announcement β allows the existential player to have a winning strategy in a b-model no matter what agent b announces at the same time, and in this case the set of responses by b will be a subset of those she had in the $a : \Omega$-model. This means that $(N_t, \rangle\!\rangle\{b\}, \beta\langle\!\langle\, \psi)$ is also a winning node for the $\exists$-player. Hence, a contradiction. See Figure 14 for an example.

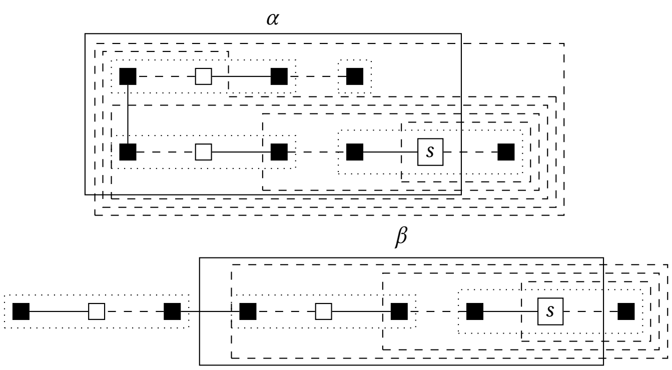

Figure 14: Set of counter-announcements (dashed rectangles) to β in the $b : \Omega$-model (below) is a subset of counter-announcements to α in the $a : \Omega$-model (above)

Case $\langle\!\langle\{b, c\}\rangle\!\rangle\psi$ is similar to the previous one.

Case $\langle\!\langle\{a\}\rangle\!\rangle\psi$: similar to $\langle\!\langle\{a, c\}\rangle\!\rangle\psi$. If a cannot get N_s^β which is isomorphic to M_s^α, then it is enough to cut a b-relation between two c-equivalence classes and 'announce' such a subset of α. It is still an a-announcement in the $b : \Omega$-model, as well as it is one of the counter-announcements in the $a : \Omega$-model (c cuts b's relation). Hence, the set of counter-announcements in the $b : \Omega$-model is the subset of counter-announcements in the $a : \Omega$-model.

Cases $\langle\!\langle\{b\}\rangle\!\rangle\psi$ and $\langle\!\langle\{c\}\rangle\!\rangle\psi$ are as the previous one. This completes the proof. □

Combining Theorems 5.2 and 5.3, we obtain the final result.

Theorem 5.4. *CAL $\not\geq$ GAL.*

Note that in case $\langle\!\langle\{a, b, c\}\rangle\!\rangle\psi$ agents a, b, and c can together enforce any configuration of a given model (up to bisimulation). This is due to the fact that the intersection of the corresponding relations is identity relation. Hence, for [1, 2]-chains $\langle\!\langle\{a, b, c\}\rangle\!\rangle\psi$ is equivalent to $\Diamond\psi$ (and $[\langle\{a, b, c\}\rangle]\psi$ is equivalent to $\Box\psi$).

Corollary 5.5. *APAL $\not\geq$ GAL.*

That APAL $\not\geq$ GAL was conjectured in [1], where it was also shown that GAL $\not\geq$ APAL. Now we combine these two results.

Theorem 5.6. *APAL and GAL are incomparable.*

We also would like to point out that the proof of GAL $\not\geq$ APAL [1] can be trivially modified to obtain that CAL $\not\geq$ APAL.

6 CONCLUDING REMARKS

We have shown that coalition announcement logic is not at least as expressive as group announcement logic (CAL $\not\geq$ GAL), and from our proof also simply follows that arbitrary public announcement logic is not at least as expressive as group announcement logic (APAL $\not\geq$ GAL). Thus we have answered two long-standing open questions in the area. These results are presented in Figure 15, in the context of the other known expressivity results and open questions for announcement logics mentioned in the introduction. In the figure, an arrow $\rightarrow$ from logic L_1 to L_2 means that $L_2 \geq L_1$. A struck-out arrow $\not\rightarrow$ means that $L_2 \not\geq L_1$, and an arrow $\overset{?}{\rightarrow}$ indicates an open problem. By EL = PAL we mean that EL $\rightarrow$ PAL and PAL $\rightarrow$ EL, and that all arrows from or to EL = PAL are from or to both EL and PAL. In future research, we would like to investigate the remaining open questions, and in particular whether GAL $\not\geq$ CAL.

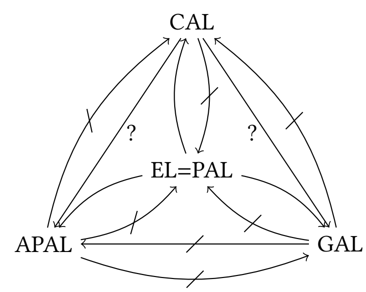

Figure 15: Relative expressivity of logics of quantified public announcements with at least 3 agents

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