Forgiving Debt in Financial Network Games

Extended Abstract

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ABSTRACT

A financial system is represented by a network, where nodes correspond to banks, and directed labeled edges correspond to debt contracts between banks. Once a payment schedule has been defined, where we assume that a bank cannot refuse a payment towards one of its lenders if it has sufficient funds, the liquidity of the system is defined as the sum of total payments made in the network. Maximizing systemic liquidity is a natural objective of any financial authority, so, we study the setting where the financial authority offers bailout money to some bank(s) or forgives the debts of others in order to maximize liquidity, and examine efficient ways to achieve this. We investigate the approximation ratio provided by the greedy bailout policy compared to the optimal one, and we study the computational hardness of finding the optimal debt removal and budget-constrained optimal bailout policy, respectively.

We also study financial systems from a game-theoretic standpoint. We observe that the removal of some incoming debt might be in the best interest of a bank, if that helps one of its borrowers remain solvent and avoid costs related to default. Assuming that a bank's well-being (i.e., utility) is aligned with the incoming payments they receive from the network, we define and analyze a game among banks who want to maximize their utility by strategically giving up some incoming payments. In addition, we extend the previous game by considering bailout payments. After formally defining the above games, we prove results about the existence and quality of pure Nash equilibria, as well as the computational complexity of finding such equilibria.

KEYWORDS

Pure Nash equilibria; Financial networks; Price of anarchy; Price of stability

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1 INTRODUCTION

A financial system comprises a set of institutions, such as banks, that engage in financial transactions. The interconnections showing the liabilities (financial obligations or debts) among the banks can be represented by a network, where the nodes correspond to banks and the edges correspond to liability relations. Each bank i has a fixed amount of external assets, e_i, (not affected by the network) which are measured in the same currency as the liabilities. A bank's *total assets*, a_i, comprise its external assets and its incoming payments, and may be used for (outgoing) payments to its lenders. If a bank's assets are not enough to cover its liabilities, that bank will be in default and the value of its assets will be decreased (e.g., by liquidation); the extent of this decrease is captured by *default costs* α (applied to external assets) and β (applied to incoming payments). The default costs essentially imply that the corresponding bank will have only a part of its total assets available for making payments.

On the liquidation day (also known as clearing), each bank in the system has to pay its debts in accordance with the following three principles of bankruptcy law (see, e.g., [5]): i) *absolute priority*, i.e., banks with sufficient assets pay their liabilities in full, ii) *limited liability*, i.e., banks with insufficient assets to pay their liabilities are in default and pay all of their assets to lenders, subject to default costs, and iii) *proportionality*, i.e., in case of default, payments to lenders are made in proportion to the respective liability. Payments that satisfy the above properties are called *clearing payments* and (perhaps surprisingly) these payments are not uniquely defined for a given financial system. However, maximal clearing payments, i.e., ones that point-wise maximize all corresponding payments, are known to exist and can be efficiently computed [12].

The total liquidity of a financial system (also referred to as systemic liquidity) is measured by the sum of payments made at clearing, and is a natural metric for the well-being of the system. Financial authorities, e.g., governments or other regulators, wish to keep the systemic liquidity as high as possible and they might interfere, if their involvement is necessary and would considerably benefit the system. For example, in the not so far past, the Greek government (among others) took loans in order to bailout banks that were in danger of defaulting, to avert collapse. In this work, we study the possibility of a financial regulating authority performing cash injections (i.e., bailouts) to selected bank(s) and/or forgiving debts selectively, with the aim of maximizing the total liquidity of the system (total money flow). We use the notion of *threat index* to capture the impact a cash-injection to a particular node would have to the liquidity of the system; we denote the threat index of bank i by μ_i. Similarly to cash injections, it is a fact that debt removal can have a positive effect on systemic liquidity. Indeed, the existence of default costs can lead to the counter-intuitive phenomenon whereby removing a debt/edge from the financial network might result in increased money flow, e.g., if the corresponding borrower avoids default costs because of the removal.

Even more surprising than the increase of liquidity by the removal of debts, is the fact that the removal of an edge from borrower b to lender l might result in l receiving more incoming payments, e.g., if b avoids default costs and there is an alternative path in the network where money can flow from b to l. This motivates the

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definition of an edge-removal game on financial networks, where banks act as strategic agents who wish to maximize their total assets and might intentionally give-up a part of their due incoming payments towards this goal. As implied earlier, removing an incoming debt could rescue the borrower from financial default, thereby avoiding the activation of default cost, and potentially increasing the lender's utility (total assets). This strategic consideration is meaningful both in the context where a financial authority performs cash injections or not. We consider the existence, quality, and computation of equilibria that arise in such games.

2 OUR CONTRIBUTION

In our work, we consider computational problems related to maximizing systemic liquidity in a financial network, when a financial authority can modify the network by appropriately removing debt, or, alternatively, by injecting cash into selected banks. We also consider financial network games where banks can choose to forgive incoming debts, if this improves their utility. The following example of a financial network serves to demonstrate relevant notions discussed in the introduction.

An example. Figure 1 provides an example of a financial network, inspired by an example in [2]. We assume that there are no default costs, i.e., $\alpha = \beta = 1$. If we denote the payment from bank i to bank j by p_{ij}, we get the following clearing payments: $p_{21} = 4.4$, $p_{32} = 3.2$, and $p_{43} = p_{45} = 1$, implying that banks v_2, v_3 and v_4 are in default. Consistent with the clearing payments above, the liquidity of the system is computed as $4.4 + 3.2 + 1 + 1 = 9.6$, while the total assets (also defined as the utility) of each bank are $a_4 = 2, a_3 = 1 + 2.2 = 3.2, a_2 = 1.2 + 3.2 = 4.4, a_1 = 4.4$ and $a_5 = 1$ respectively. The threat indexes are computed as follows: $\mu_1 = \mu_5 = 0$, $\mu_2 = 1 + \mu_1$, $\mu_3 = 1 + \mu_2$, and $\mu_4 = 1 + \frac{1}{2}\mu_3 + \frac{1}{2}\mu_5$, implying that $\mu_3 = \mu_4 = 2$ and $\mu_2 = 1$.

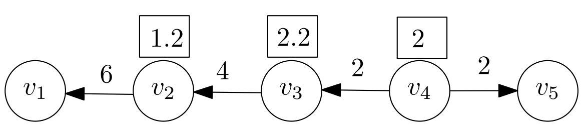

Figure 1: A simple financial network. Nodes correspond to banks, edges are labelled with the respective liabilities, while external assets are in a rectangle above the relevant banks.

We show how to compute the optimal cash injection policy in polynomial time when there are no default costs, by solving a linear program; the problem is NP-hard when non-trivial default costs apply. As our LP-based algorithm requires knowledge of the available budget and leads to non-monotone payments, we study the approximation ratio of a greedy cash injection policy based on the threat indexes. Regarding debt removal, we prove that finding the set of liabilities whose removal maximizes systemic liquidity is NP-hard. We also note that the objective of systemic solvency, i.e., guaranteeing that all banks are solvent, can be trivially achieved by removing all edges. However, adding a liquidity target, or a target on the amount of deleted liabilities, makes this problem hard.

Regarding edge-removal games, with or without bailout, we study the existence and the quality of Nash equilibria, while also addressing computational complexity questions. Apart from arguing about well-established notions, such as the Price of Anarchy and the Price of Stability, we introduce the notion of the Effect of Anarchy (Stability, respectively) as a new measure on the quality of equilibria in this setting.

2.1 Related Work

Our model is based on the seminal work of Eisenberg and Noe [5] who introduced a widely adopted model for financial networks, assuming debt-only contracts and proportional payments. This was later extended by Rogers and Veraart [12] to allow for default costs. Additional features have been since introduced, see e.g., [15] and [10]. We follow the model of Eisenberg and Noe and consider proportional payments; we note that a recent series of papers introduced different payment schemes [1, 9, 10].

When the financial regulator has available funds to bailout each bank of the network, Jackson et al. [8] characterize the minimum bailout budget needed to ensure systemic solvency and prove that computing it is an NP-hard problem. When the financial authority has limited bailout budget, Demange [2] proposes the threat index as a means to determine which banks should receive cash during a default episode and suggests a greedy algorithm for this process. Egressy and Wattenhofer [4] focus on how central banks should decide which insolvent banks to bailout and formulate corresponding optimization problems. Dong et al. [3] introduce an efficient greedy-based clearing algorithm for an extension of the Eisenberg-Noe model, while also studying bailout policies when banks in default have no assets to distribute. We note that the problem of injecting cash (as subsidies) in financial networks has been studied (in a different context) in microfinance markets [7].

Further work includes [13] that considers the incentives banks might have to approve the removal of a set of liabilities forming a directed cycle in the financial network, while [14] considers the complexity of finding clearing payments when CDS contracts are allowed. In a similar spirit, [6] studies the clearing problem from the point of view of irrationality and approximation strength, while [11] studies which banks are in default, and how much of their liabilities these defaulting banks can pay.

3 CONCLUSIONS

We considered problems arising in financial networks, when a financial authority wishes to maximize the total liquidity either by injecting cash or by removing debt. We also studied the setting where banks are rational strategic agents that might prefer to forgive some debt if this leads to greater utility, and we analyzed the corresponding games with respect to properties of Nash equilibria. In that context, we also introduced the notion of the Effect of Anarchy (Stability, respectively) that compares the liquidity in the initial network to that of the worst (best, respectively) Nash equilibrium.

Our work leaves some interesting problems unresolved. Given the computational hardness of some of the optimization problems, it makes sense to consider approximation algorithms. From the game-theoretic point of view, one can also consider the problems from a mechanism design angle, i.e., to design incentive-compatible policies where banks weakly prefer to keep all incoming liabilities.

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