A Study of Nash Equilibria in Multi-Objective Normal-Form Games

JAAMAS Track

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ABSTRACT

We present a detailed analysis of Nash equilibria in multi-objective normal-form games, which are normal-form games with vectorial payoffs. Our approach is based on modelling each player's utility using a utility function that maps a vector to a scalar utility. For mixed strategies, we can apply the utility function before calculating the expectation of the payoff vector as well as after, resulting in two distinct optimisation criteria. We show that when computing the utility from the expected payoff, a Nash equilibrium can be guaranteed when players have quasiconcave utility functions. In addition, we show that when players have quasiconvex utility functions, pure strategy Nash equilibria are equal under both optimisation criteria. We extend this to settings where some players optimise for one criterion, while others optimise for the second. We combine these results and formulate an algorithm that computes all pure strategy Nash equilibria given quasiconvex utility functions.

KEYWORDS

Multi-objective; Game theory; Nash equilibrium

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1 INTRODUCTION

To function effectively in complex environments, artificial agents must be equipped to deal with numerous challenges in their decision making. A first challenge is the presence of other agents that interact with the environment and may influence the outcome of different actions. Moreover, in many real-world applications, multiple conflicting objectives need to be optimised simultaneously, necessitating exploration of the trade-offs between them [8]. To study such complex scenarios, we turn to multi-objective games that combine insights from game theory and multi-objective decision making.

We focus on Multi-Objective Normal-Form Games (MONFGs) [1], a generalisation of (single-objective) Normal-Form Games (NFG). We further take a utility-based approach which assumes the existence of a utility function for each agent that maps a vector to a scalar utility [7]. In the case of a mixed strategy, agents can apply their utility function either before calculating the expectation of the payoff vector or after, resulting in two distinct optimisation criteria. The former, referred to as the *expected scalarised returns* (ESR) criterion, is applicable when optimising for the outcome of a single strategy execution. The latter is referred to as the *scalarised expected returns* (SER) criterion and is applicable when optimising the utility of repeated plays of the game. It can be shown that the choice of optimality criterion influences what strategies are optimal [12] and presents different theoretical guarantees [6]. Consider for example the payoffs $(2, 0)$ and $(0, 2)$ and the product utility function $u(p_1, p_2) = p_1 \cdot p_2$. For a uniform mixture over the two payoffs, the utility of the expected payoff is equal to one but is different than the expected utility of the payoffs which is equal to zero. See [5] for a survey of multi-objective multi-agent decision making.

Motivated by recent work in these games and their practical relevance, we perform an in-depth study of Nash equilibria under both optimisation criteria. The full version of this work with additional motivation and proofs can be found in [9].

2 THEORETICAL CONTRIBUTIONS

We present novel results on the existence of equilibria in MONFGs as well as the relations between ESR and SER. In addition, we study blended settings where players may optimise for different criteria.

2.1 Nash Equilibrium Existence

Previous work has shown that Nash equilibria need not exist in MONFGs with nonlinear utility functions under the SER criterion [6]. Given these negative results, it is crucial to study what conditions are necessary or sufficient to guarantee existence. In Theorem 2.1 we contribute the first existence guarantee for this setting by restricting the class of utility functions that players may use.

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Theorem 2.1. *For any finite MONFG under SER where players have continuous quasiconcave utility functions, a Nash equilibrium is guaranteed to exist.*

The proof relies on the reduction of the MONFG to an equivalent single-objective game with an infinite number of pure strategies [11]. In such games, the guarantee for quasiconcave utility functions is known which allows us to introduce it for MONFGs. We note that quasiconcavity is a relaxation of concavity and has been considered a suitable representation of human preferences [3].

Recent follow-up work showed that it is also always possible to transform an infinite game into an MONFG [10]. The resulting equivalence between MONFGs and infinite games is referred to as *pure strategy equivalence* and enables the exchange of theoretical and algorithmic results.

Contrary to the positive result for quasiconcave utility functions, we find that it is possible to construct games without Nash equilibria when restricting players to strictly convex utility functions. This further precludes existence guarantees under convex or quasiconvex utility functions.

2.2 Equilibrium Relations

It is known that an MONFG under ESR can be reduced to a (single-objective) NFG, called the *trade-off game.* As NFGs have been well-studied both theoretically and algorithmically, it is important to determine when the ESR and SER criteria correspond. However, here we obtain the negative result that, in general, both the number of equilibria as well as the equilibria themselves may be different. We formally show this by constructing games with the desired properties [9].

Theorem 2.2. *Consider an MONFG with at least one Nash equilibrium under SER and ESR. The sets of Nash equilibria may have different cardinality and be disjoint.*

The main issue that leads to Theorem 2.2 is that a function applied to the expectation of a random variable is not generally equal to the expectation of the function applied to the random variable. We can sidestep this problem by focusing on pure strategy Nash equilibria where players deterministically play a single action. Here, we leverage a generalised version of Jensen's inequality to show that when restricting the utility functions to be quasiconvex, pure strategy Nash equilibria under ESR and SER correspond.

Theorem 2.3. *For any MONFG where players have quasiconvex utility functions, the pure strategy Nash equilibria are equal under SER and ESR.*

2.3 Blended Settings

Finally, we consider a novel setting for MONFGs where some players optimise for SER and others for ESR. One example of such a setting is a job market where an employer repeatedly offers positions with the same terms and conditions to different job seekers who play the game only once. Here, the employer may care about the utility of the expected payoff they get from shaping their team, and balancing the talents of some employees with others, while each job seeker cares about maximising their expected utility from the job interview. We refer to these settings as *blended* settings. A Nash equilibrium is defined as a joint strategy where no player can unilaterally deviate and improve on their respective optimisation criterion. We find that Theorem 2.3 is straightforward to extend to this setting.

Theorem 2.4. *Consider an MONFG where each player has a quasiconvex utility function. The set of pure strategy Nash equilibria in the trade-off game is equal to the set of pure strategy Nash equilibria in any blended setting.*

Note that Theorem 2.4 holds true irrespective of which players optimise for ESR and SER and even how many optimise for either criterion. Furthermore, we may exploit Theorems 2.3 and 2.4 for a novel Nash equilibrium existence guarantee. Concretely, if the MONFG has quasiconvex utility functions and its trade-off game belongs to a class of games with pure strategy Nash equilibrium guarantees, this guarantees a pure strategy Nash equilibrium in the MONFG as well. Potential games are a well-known example of such a class of NFGs [4].

3 ALGORITHMIC IMPLICATIONS

The contributions presented here lead to a straightforward algorithm for computing all pure strategy equilibria in any MONFG, given that quasiconvex utility functions are assumed[1]. The algorithm can be summarised by its two steps. First, it is needed to compute the trade-off game. This can be done by applying the utility function of each player to their vectorial payoff matrices. The resulting (single-objective) payoff matrices can then be shown to be equivalent to the MONFG under ESR [6].

Second, the set of pure strategy Nash equilibria needs to be computed in the trade-off game. A naive approach is to enumerate all joint strategies and verify for each player that no gain is possible by deviating to a different strategy. The joint strategies that remain are then the pure strategy Nash equilibria in the trade-off game and, due to the shown theorems, also equilibria under ESR, SER and any blended setting. We note that there exist alternative methods that compute the set of pure strategy equilibria in NFGs with improved computational complexity [2]. Additionally, the second step of the algorithm can be adapted to find a sample pure strategy Nash equilibrium rather than the full set.

Finally, a similar approach is possible for computing the pure strategy equilibria in an MONFG with arbitrary utility functions. Concretely, one can enumerate all joint strategies and subsequently verify for each player that their strategy is a best response. However, verifying a best response requires performing a global optimisation subroutine. The benefit of the proposed algorithm is that it avoids doing such costly optimisations by first reducing to a trade-off NFG.

4 FUTURE WORK

For future work, we focus on the interplay between theory and practice. First, we aim to obtain more comprehensive existence guarantees and develop criteria for identifying situations where equilibria are guaranteed to not exist. This leads to the second goal, which is to design algorithms that leverage this knowledge to learn or compute (approximate) equilibria. Finally, it may be interesting to investigate the application of these techniques to real-world problems, such as resource allocation, traffic management, or market competition.

[1]We provide an implementation of this algorithm in https://github.com/wilrop/moqups.

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