Indivisible Participatory Budgeting with Multiple Degrees of Sophistication of Projects

Extended Abstract

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ABSTRACT

Indivisible participatory budgeting (PB) is a framework that aggregates the preferences of voters to decide the distribution of budget among a set of projects. The existing literature assumes that each project has only one possible cost. In this work, we let each project have a set of permissible costs, each reflecting a possible degree of sophistication of the project. Each voter approves a range of costs for each project, by giving an upper and lower bound on the cost that she thinks the project deserves. We prove that the existing positive results can also be extended to our framework where a project has several permissible costs, and also present new computational and axiomatic results.

KEYWORDS

Indivisible Participatory Budgeting; Axioms; Complexity

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1 INTRODUCTION

Participatory budgeting (PB) is a democratic voting paradigm that aggregates the opinions of citizens to decide how to fund the public projects. It is being implemented in hundreds of municipalities and institutions in various countries throughout the world [8, 17, 18]. PB is classified into two models: divisible PB and indivisible PB.

Divisible PB assumes that the costs of the projects are totally flexible and any amount can be allocated to each of them [1, 2, 6, 11, 22]. The existing work on indivisible PB, on the other hand, assumes that every project has a fixed cost that is to be allocated if the project is selected [5, 13, 14, 16, 19, 20, 23].

However, many times in real-world, each project can be implemented upto different levels of sophistication. For example, a shopping mall can be constructed in different sizes and magnitudes, or a building can be constructed with different materials. That is, in our model, each project has a *set of permissible costs*. Each cost in this set corresponds to a degree of sophistication of the project. The PB rule must choose the projects that are to be funded and also a permissible cost for each of them.

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The preference elicitation methods typically studied in PB include approval votes, ordinal votes, and cardinal votes [4, 5, 10, 12, 15, 16, 19, 21, 23]. However, the preferences and utilities of the voters in PB are much more complex. This propels the need to devise preference models specific to PB context, as pointed out by Aziz and Shah [7]. Our paper narrows this gap by introducing *ranged approval votes*, which strictly generalize the approval votes. Each voter reports a lower bound and an upper bound on the cost that she thinks each project deserves. All the bounds are initially set to 0 by default. Voting proceeds in two steps. In the first step, the voter starts by approving the projects she likes. For these approved projects, only the upper bounds will automatically change to the highest permissible cost. In the second step, the voter may *optionally* change bounds for some of these approved projects, if she wishes to have a say on the amount they deserve.

Goel et al. [14] study a special case of the ranged approval votes where lower bounds are always 0. Talmon and Faliszewski [23] study another special case where every project has only one cost.

2 MODEL

Let b be the budget, $N = \{1, \ldots, n\}$ be the set of voters and P be the set of m projects. Each project $j \in P$ has t_j possible degrees of sophistication captured by the set $\mathrm{D}(P_j) = \{P_j^0, P_j^1, \ldots, P_j^{t_j}\}$. The cost of each degree P_j^t is indicated by c_j^t. We assume that c_j^0 is zero for all $j \in P$ and it corresponds to not funding the project j. Let $\mathcal{D}$ denote the set of all the possible degrees of all projects, or in other words, $\mathcal{D} = \bigcup_{j \in P} \mathrm{D}(P_j)$. We denote the cost of a set $S \subseteq \mathcal{D}$, $\sum_{P_j^t \in S} c_j^t$, by $\mathrm{c}(S)$. Each voter $i \in N$ reports for every project j, a lower bound $l_i(j)$ and an upper bound $h_i(j)$ such that $l_i(j), h_i(j) \in \{c_j^0, \ldots, c_j^{t_j}\}$ and $l_i(j) \le h_i(j)$.

Given a subset $S \subseteq \mathcal{D}$, $S(j)$ denotes $S \cap \mathrm{D}(P_j)$ and $c^S(j)$ denotes $\mathrm{c}(S(j))$. A subset $S \subseteq \mathcal{D}$ is ***valid*** if $\mathrm{c}(S) \le b$ and $|S(j)| = 1$ for every $j \in P$. Let $\mathcal{V}$ be the collection of all the valid subsets. The objective of a PB rule R is to output a valid subset $S \in \mathcal{V}$ for a given instance $I = \langle N, \mathcal{D}, \mathrm{c}, b, (l_i(j), h_i(j))_{i,j} \rangle$.

2.0.1 ***The PB Rules.*** We study utilitarian rules, i.e., given a utility function u, the rule outputs a valid set of projects that maximizes $\sum_{i \in N} u_i(S)$. We say a set S is selected if it maximizes the total utility. We say a project $P_j^t \in \mathcal{D}$ ***wins*** under a PB rule R if it belongs to some set that can be selected under R. Let $R(I)$ be the collection of all the projects that win under the PB rule R, given an instance I. We define four PB rules each with a different utility function:

(1) $\boldsymbol{R_{|S|}}$: $u_i(S) = |j \in P : l_i(j) \le c^S(j) \le h_i(j)|$.
(2) $\boldsymbol{R_{\mathrm{c}(S)}}$: $u_i(S) = \sum_{j : l_i(j) \le c^S(j) \le h_i(j)} c^S(j)$.

RULES	Exact Computation	Approximation	FPT
$R_{\lvert S\rvert}$	P	N.A.	N.A.
$R_{c(S)}, R_{\widehat{c(S)}}$	Weakly NP-hard	FPTAS	w.r.t. δ w.r.t. m
$R_{\lVert\,\rVert}$	NP-hard	FPTAS parameterized by γ	w.r.t. δ w.r.t. m

Table 1: Computational complexity of PB rules

PROPERTIES\RULES	$R_{\lvert S\rvert}$	$R_{c(S)}$	$R_{\widehat{c(S)}}$	$R_{\lVert\,\rVert}$
Shrink-resitance	✓	✓	✓	✓
Discount-proofness	✓	×	×	×
Degree-efficiency	×	×	✓	×
Lower bound-sensitivity	×	×	×	✓
Upper bound-sensitivity	✓	✓	×	✓
Range-unanimity	✓	×	×	✓
Range-abidingness	✓	×	×	✓
Range-convergingness	✓	✓	✓	✓

Table 2: Results for budgeting axioms in Section 4

(3) $R_{\widehat{c(S)}}$: With slight abuse of notation, $u_i(S) = \sum_{j\in P} u_i(S,j)$, where $u_i(S,j)$ is defined as follows:

$$u_i(S,j) = \begin{cases} 0 & c^S(j) < l_i(j) \\ c^S(j) & l_i(j) \le c^S(j) \le h_i(j) \\ h_i(j) & \text{otherwise} \end{cases}$$

(4) $R_{\lVert\,\rVert}$: With slight abuse of notation, $u_i(S) = \sum_{j\in P} u_i(S,j)$, where $u_i(S,j)$ is defined as follows:

$$u_i(S,j) = \begin{cases} c^S(j) - l_i(j) & c^S(j) < l_i(j) \\ 0 & l_i(j) \le c^S(j) \le h_i(j) \\ h_i(j) - c^S(j) & \text{otherwise} \end{cases}$$

The first two utilities are the natural extensions to notions in [23]. Former reflects that the voter is happy as long as the cost allocated is acceptable to her, whereas the latter reflects that among the acceptable costs, the voter is happier from the higher cost. The third notion assumes that the voter is indifferent to more amount being spent, whereas the fourth one assumes that farther the allocated cost is from the acceptable range, lesser is the utility.

3 COMPUTATIONAL COMPLEXITY

We present in Table 1, the complexity of computing a valid subset that maximizes the total utility, for each of our rules. Here, the parameter δ denotes scalable limit [21], which is $\frac{\max_{P_j^t\in\mathcal{D}} c_j^t}{GCD(c_1^1,\ldots,c_m^{t_m},b)}$ and γ denotes the novel parameter q_m/q_σ where $q_m = \max_{P_j^t\in\mathcal{D}} q_j^t$ with $q_j^t = \sum_{i:c_j^t<l_i(j)} (l_i(j) - c_j^t) + \sum_{i:h_i(j)<c_j^t} (c_j^t - h_i(j))$, and $q_\sigma = \sum_{j\in P} \min_{t\in D(P_j)} q_j^t$.

4 BUDGETING AXIOMS

Though there is an enormous amount of work on the axiomatic study in PB [3, 5, 9, 16, 21, 23], the uniqueness of our model motivates us to propose new axioms that are explicitly relevant here.

4.0.1 Monotonicity Axioms. Shrink-resistance requires that if the reported bound moves closer to the selected degree, the same degree must continue to be selected. *Discount-proofness* ensures that if a selected degree becomes more affordable, it continues to win.

Definition 1 (Shrink-resistance). *For any instance I, a voter i, any $j \in P$, it holds that a set S selected under R continues to be selected even if $l_i(j)$ and $h_i(j)$ are shifted closer to $c^S(j)$.*

Definition 2 (Discount-proofness). *For any instance I, any project $j \in P$, and a set S that is selected, S continues to be selected if $c^S(j)$ is decreased by 1.*

4.0.2 Efficiency Axioms. . The axiom *degree-efficiency* requires that if the selected degree of a project can be increased without disturbing the costs allocated to other projects, then the rule must do so. *Lower-bound* and *upper-bound sensitivities* essentially mean that an outcome whose selected degrees are closest to the bounds reported by all the voters (if not within them) is preferred over the others.

Definition 3 (Degree-efficiency). *For any instance I, any $j \in P$, and any selected set S, we have $k > S(j) \Longrightarrow c(S) - c^S(j) + c_j^k > b$.*

Definition 4 (Lower bound-sensitivity). *For any instance I, any project $j \in P$, and any two valid set S, S' such that for every voter $i \in N$ we have $c^S(j) < c^{S'}(j) < l_i(j)$, S is not selected under R.*

Definition 5 (Upper bound-sensitivity). *For any instance I, any $j \in P$, and any two valid sets S, S' such that for every voter $i \in N$ we have $c^S(j) > c^{S'}(j) > h_i(j)$, S is not selected under R.*

4.0.3 Unanimity Axioms. Let τ_j represent the intersection of intervals reported by all the voters for project j and $\overline{\tau_j} = \max(\tau_j)$. *Range-unanimity* requires that if allocating $\bar{\tau_j}$ to every j is feasible, then the rule must do so. *Range-abidingness* requires that the selected cost of a project cannot be higher than $\bar{\tau_j}$. Note that the above both axioms do not imply each other, though they seem closely related. *Range-convergingness* requires that increasing b moves the selected cost of at least one project j closer to τ_j.

Definition 6 (Range-unanimity). *For any instance I, whenever $\sum_{j\in P} \overline{\tau_j} \le b$, the set $\{P_j^t : j \in P,\ c_j^t = \overline{\tau_j}\}$ is selected.*

Definition 7 (Range-abidingness). *For any instance I, a project $j \in P$, and a selected set S, we have $\tau_j \ne \emptyset \implies c^S(j) \le \overline{\tau_j}$.*

Definition 8 (Range-convergingness). *For any instance I, a selected set S, and a set $S' \ne S$ selected on increasing the budget, there exists some project $j \in P$ such that: $c^S(j) \notin \tau_j \implies |c^S(j) - \underline{\tau_j}| > |c^{S'}(j) - \underline{\tau_j}|$.*

5 SUMMARY

We introduce the model with multiple degrees of sophistication of projects and ranged approval votes. We generalize two existing utility notions [23] and introduce two other novel notions. We strengthen all the existing positive results [23] and also present new parameterized tractability results. Each of these results can be generalized to the model with cardinal utilities, but we confine to ranged approval votes due to their practical relevance and simplicity. Finally, we propose novel budgeting axioms and present axiomatic analysis (Table 2). Note that the PB rule $R_{\lVert\,\rVert}$ we introduced satisfies as many axioms as any simple approval PB rule satisfies, thus making it a very good choice for ranged approval votes.

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