Diffusion Multi-unit Auctions with Diminishing Marginal Utility Buyers

Extended Abstract

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ABSTRACT

We consider an auction design problem where a seller sells multiple homogeneous items to a set of connected buyers. Each buyer only knows the buyers she directly connects with, and the seller initially only connects to a few buyers. Our goal is to design an auction to incentivize the buyers who are aware of the market to invite their neighbors to join the auction. Meanwhile, the auction should also guarantee that the seller never runs a deficit. In this paper, we design the very first multi-unit diffusion auction that satisfies all these properties for buyers with diminishing marginal utility.

KEYWORDS

Auction Design; Invitation Incentive; Social Networks.

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1 INTRODUCTION

Multi-unit auctions refer to auctions where multiple homogeneous items are available for sale in a single auction. A common feature of these markets is that the participants are known in advance, and the seller can hold the classic Vickrey-Clarke-Groves (VCG) mechanism [2, 3, 15] to get good social welfare and revenue. To further improve social welfare and revenue, an intuitive method is to advertise the sale to involve more buyers. However, the seller normally needs to pay for the advertisements and if the ads can't attract enough valuable buyers, the seller's revenue may decrease. To solve this problem, diffusion mechanism design has been proposed in recent years [7, 10, 20, 21]. Diffusion mechanisms utilize the participants' connections to attract more buyers. Every participant who is aware of the market has incentives to invite all her neighbors. The rewards to buyers who do effective invitations are carefully designed to ensure the seller has revenue improvement.

Li et al. [10] first modeled the single-item diffusion auction setting and proposed the information diffusion mechanism (IDM) to incentivize buyers to invite each other. GIDM [22] and DNA-MU [5] extended the setting to multi-unit-supply and unit-demand cases.

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However, an error in the proofs of GIDM has been found [14]. In the full version of this paper, we show that DNA-MU is also problematic [11]. Thus, there is no satisfactory diffusion auction mechanism even in multi-unit-supply and unit-demand settings yet. Takanashi et al. [14] also studied the multi-unit-demand settings but they focus on efficiency approximability and cannot guarantee the seller is profitable.

In this paper, we design the very first incentive-compatible diffusion auction in multi-unit-supply and multi-unit-demand settings that has satisfactory revenue guarantees. We propose the layer-based diffusion mechanism (LDM). LDM first converts the given network into its breadth-first search tree and computes allocations and payments layer by layer in the tree. When computing for one layer, all potential competitors of this layer are removed to avoid buyers' misreports. By doing so, we also removed many buyers who are not competitors of the layer, which will reduce the social welfare and revenue.

In addition to extending diffusion auctions to more general settings, single-unit diffusion auctions have also been extensively studied. For single-unit diffusion auctions, following IDM, Li et al. [9] gave a general auction class; Zhang et al. [17] proposed a fairer reward scheme; Li et al. [8] further characterized the conditions to achieve incentive compatibility. Beyond auctions, diffusion incentives can also be introduced to other classical settings, such as the buyer-centric market for procurement [12, 13], redistribution mechanisms [18], house allocation [6, 16], two-sided matching [1] and cooperative games [19]. Comprehensive surveys on diffusion mechanism design are given in [4, 20, 21].

2 THE MODEL

We consider an auction where a seller s sells $\mathcal{K} \geq 1$ homogeneous items via a network. In addition to the seller, the social network consists of n potential buyers denoted by $N = \{1, \cdots, n\}$. Each $i \in N$ has a private marginal decreasing utility function for the $\mathcal{K}$ items which is denoted by a value vector $v_i = (v_i^1, \cdots, v_i^{\mathcal{K}})$ where $v_i^1 \geq v_i^2 \geq \cdots \geq v_i^{\mathcal{K}} \geq 0$. Let $v_i(m) = \sum_{k=1}^{m} v_i^k$ be the valuation of i for receiving $m \geq 1$ units and assume $v_i(0) = 0$. Each buyer $i \in N$ has a set of neighbors $r_i \subseteq N \cup \{s\}$ and i does not know the existence of the others except for r_i. The seller is also only aware of her neighbors r_s initially. Let $\theta_i = (v_i, r_i)$ be the *type* of buyer $i \in N$ and $\theta = (\theta_1, \cdots, \theta_n) = (\theta_i, \theta_{-i})$ be the type profile of all buyers where θ_{-i} is the type profile of all buyers except for i. We want to design auction mechanisms that ask each buyer to report her valuations and invite her neighbors to join the mechanism. This is mathematically modeled by reporting her type. Let $\hat{\theta}_i = (\hat{v}_i, \hat{r}_i)$ be buyer i's type report where $\hat{r}_i \subseteq r_i$ because i can not invite someone she does not know. Let $\hat{\theta} = (\hat{\theta}_1, \cdots, \hat{\theta}_n) = (\hat{\theta}_i, \hat{\theta}_{-i})$ be the report type profile of all buyers. Given a report type profile $\hat{\theta}$, if a buyer i and the seller s are connected, then we say i is *valid* in the auction. Let $Q(\hat{\theta})$ be the set of all valid buyers given $\hat{\theta}$.

A general auction mechanism consists of an *allocation policy* $\pi = (\pi_i)_{i\in N}$ and a *payment policy* $p = (p_i)_{i\in N}$. Given a report type profile $\hat{\theta}$, $\pi_i(\hat{\theta}) \in \{0, 1, \cdots, \mathcal{K}\}$ is the number of items i receives and $\sum_{i\in N} \pi_i(\hat{\theta}_i) \leq \mathcal{K}$. $p_i(\hat{\theta}) \in \mathbb{R}$ is the payment that i pays to the mechanism. If $p_i(\hat{\theta}) < 0$, then i receives $|p_i(\hat{\theta})|$ from the mechanism. Given $\hat{\theta}$, for mechanism (π, p), the *social welfare* is defined as $\sum_{i=1}^{n} v_i(\pi_i(\hat{\theta}))$ and the *revenue* is $\sum_{i=1}^{n} p_i$. A *diffusion auction mechanism* (π, p) is an auction mechanism that only runs among valid buyers $Q(\hat{\theta})$ and the output is independent of buyers in $N \setminus Q(\hat{\theta})$. Given a buyer i of type $\theta_i = (v_i, r_i)$ and a report type profile $\hat{\theta}$, the *utility* of i under a diffusion auction mechanism (π, p) is defined as $u_i((\pi, p), \hat{\theta}) = v_i(\pi_i(\hat{\theta})) - p_i(\hat{\theta})$.

Definition 2.1. A diffusion auction mechanism (π, p) is *individually rational* (IR) if $u_i((\pi, p), (v_i, \hat{r}_i), \hat{\theta}_{-i}) \geq 0$ for all $i \in N$, all $\hat{r}_i \subseteq r_i$, and all $\hat{\theta}_{-i}$.

Definition 2.2. A diffusion auction mechanism (π, p) is *incentive compatible* (IC) if $u_i((\pi, p), \theta_i, \hat{\theta}_{-i}) \geq u_i((\pi, p), \hat{\theta}_i, \hat{\theta}_{-i})$ for all $i \in N$, all $\hat{\theta}_i$ and all $\hat{\theta}_{-i}$.

Given a report type profile $\hat{\theta}$, an undirected graph $\mathcal{G}(\hat{\theta}) = (V(\hat{\theta}) \cup \{s\}, E(\hat{\theta}))$ can be constructed where $V(\hat{\theta}) = \{i | i \in Q(\hat{\theta})\}$. For each $i \in V(\hat{\theta}) \cup \{s\}$ and each $j \in \hat{r}_i$, there is an edge $(i, j) \in E(\hat{\theta})$. For each buyer $i \in Q(\hat{\theta})$, let $d_i(\hat{\theta})$ be the shortest distance in $\mathcal{G}(\hat{\theta})$ from the seller s to i. Let $\mathcal{L}_d(\hat{\theta})$ be the set of valid buyers whose shortest distance to the seller is d, i.e., $\mathcal{L}_d(\hat{\theta}) = \{i | i \in Q(\hat{\theta}), d_i(\hat{\theta}) = d\}$. We also call $\mathcal{L}_d(\hat{\theta})$ the layer d. Let $\mathcal{L}_{<l}(\hat{\theta}) = \bigcup_{1\leq i<l} \mathcal{L}_i(\hat{\theta})$ and $\mathcal{L}_{>l}(\hat{\theta}) = \bigcup_{i>l} \mathcal{L}_i(\hat{\theta})$

3 LAYER-BASED DIFFUSION MECHANISM

In this section, we design a mechanism called layer-based diffusion mechanism(LDM), for buyers with diminishing marginal utility. In LDM, we first transform graph $\mathcal{G}(\hat{\theta})$ into its breadth-first search tree (BFS tree) $\mathcal{T}^{BFS}(\hat{\theta})$. Then, LDM prioritizes buyers by the layers and decides the allocations and payments layer by layer. When computing the allocations of layer l, we first fix the allocations of buyers in lower layers $\mathcal{L}_{<l}(\hat{\theta})$ and remove the buyers $\mathcal{R}_l(\hat{\theta}) \subseteq \mathcal{L}_{>l}(\hat{\theta})$ from the higher layers. $\mathcal{R}_l(\hat{\theta})$, which will be defined later, should include the buyers below layer l who are *potential competitors* of layer l. After $\mathcal{R}_l(\hat{\theta})$ is removed, in the remaining buyers, we use a VCG-like policy to determine the allocations and payments of layer l. For any buyer $i \in V(\hat{\theta})$, let $C_i(\hat{\theta})$ be i's children in $\mathcal{T}^{BFS}(\hat{\theta})$. Let l^{max} denote the total number of layers in $\mathcal{T}^{BFS}(\hat{\theta})$. Algorithm 1 formally defines LDM.

Algorithm 1 Layer-based Diffusion Mechanism(LDM)

Require: A report profile $\hat{\theta}$;
Ensure: $\pi(\hat{\theta})$ and $p(\hat{\theta})$;

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Construct the BFS tree T^BFS(θ̂) of graph G(θ̂).
Initialize K^remain = K;
for l = 1, 2, ···, l^max do
    Compute the following constrained optimization problem
    and let π^l(θ̂) be the optimal solution.
        max_{π(θ̂)} SW_{-R_l}(θ̂) = Σ_{i∈Q(θ̂)\R_l(θ̂)} v̂_i(π_i(θ̂))
        s.t. When l ≠ 1, ∀q < l, ∀j ∈ L_q, π_j(θ̂) = π_j^q(θ̂)
    for i ∈ L_l(θ̂) do
        Set D_i = R_l(θ̂) ∪ C_i(θ̂) ∪ {i}
        Compute the following constrained optimization problem:
            max_{π(θ̂)} SW_{-D_i}(θ̂) = Σ_{j∈Q(θ̂)\D_i} v̂_j(π_j(θ̂))
            s.t. When l ≠ 1, ∀q < l, ∀j ∈ L_q, π_j(θ̂) = π_j^q(θ̂)
        Set π_i(θ̂) = π_i^l(θ̂);
        if π_i^l(θ̂) ≠ 0 then
            K^remain = K^remain − π_i^l(θ̂);
            p_i(θ̂) = SW_{-D_i}(θ̂) − (SW_{-R_l}(θ̂) − v̂_i(π_i^l(θ̂)));
        else
            p_i(θ̂) = SW_{-D_i}(θ̂) − SW_{-R_l}(θ̂);
        end if
    end for
    if K^remain = 0 then
        Set π_i(θ̂) = p_i(θ̂) = 0, ∀k > l, ∀i ∈ L_k(θ̂)
        break
    end if
end for
Return π_i(θ̂) and p_i(θ̂) for each buyer i.
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In LDM, $\mathcal{R}_l(\hat{\theta})$ contains all buyers who potentially have positive utilities. We divide those buyers into two parts: buyers who diffuse information to potential winners and buyers who are potential winners. In our design, for each $i \in \mathcal{L}_l(\hat{\theta})$, the first part corresponds to $C_i^{\mathcal{P}}(\hat{\theta}) = \{j | j \in C_i(\hat{\theta}), C_j(\hat{\theta}) \neq \emptyset\} \subseteq C_i(\hat{\theta})$ who are the children of i who have also children. The second part corresponds to $C_i^{\mathcal{W}}(\hat{\theta})$ who are the top $\mathcal{K} + \mu - |C_i^{\mathcal{P}}(\hat{\theta})|$ ranked buyers in $C_i(\hat{\theta}) \setminus C_i^{\mathcal{P}}(\hat{\theta})$ according their valuation reports for the first unit, where μ is a constant with $\max_{i\in N} |C_i^{\mathcal{P}}(\hat{\theta})| \leq \mu$. We assume μ is prior information to the seller which only depends on the network structure. Taking the union of all these sets, the final definition of the removed set $\mathcal{R}_l(\hat{\theta})$ is $\left(\bigcup_{i\in \mathcal{L}_l(\hat{\theta})} C_i^{\mathcal{W}}(\hat{\theta}) \cup C_i^{\mathcal{P}}(\hat{\theta})\right) \cup \left(\bigcup_{l+2\leq d\leq l^{max}} \mathcal{L}_d(\hat{\theta})\right)$. Theorem 3.1 summarizes all the properties of LDM. The proof is given in the full version of this paper [11].

THEOREM 3.1. *The LDM is individually rational (IR) and incentive compatible (IC). The social welfare and revenue of LDM are no less than the social welfare and revenue of running the VCG mechanism among r_s.*

4 CONCLUSIONS

We designed the layer-based diffusion mechanism (LDM) for multi-unit diffusion auctions with diminishing marginal utility buyers. Although LDM relies on the prior knowledge of μ, it is the very first multi-unit diffusion auction that satisfies all desirable properties. In future works, it is possible to refine the design of $\mathcal{R}_l(\hat{\theta})$ to remove μ.

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REFERENCES

[1] Sung-Ho Cho, Taiki Todo, and Makoto Yokoo. 2022. Two-Sided Matching over Social Networks. In *31st International Joint Conference on Artificial Intelligence, IJCAI 2022*. International Joint Conferences on Artificial Intelligence, 186–193.

[2] Edward H Clarke. 1971. Multipart pricing of public goods. *Public choice* (1971), 17–33.

[3] Theodore Groves. 1973. Incentives in teams. *Econometrica: Journal of the Econometric Society* (1973), 617–631.

[4] Yuhang Guo and Dong Hao. 2021. Emerging methods of auction design in social networks. *arXiv preprint arXiv:2108.00381* (2021).

[5] Takehiro Kawasaki, Nathanaël Barrot, Seiji Takanashi, Taiki Todo, and Makoto Yokoo. 2020. Strategy-Proof and Non-Wasteful Multi-Unit Auction via Social Network. In *The Thirty-Fourth AAAI Conference on Artificial Intelligence, AAAI 2020*. 2062–2069.

[6] Takehiro Kawasaki, Ryoji Wada, Taiki Todo, and Makoto Yokoo. 2021. Mechanism design for housing markets over social networks. In *Proceedings of the 20th International Conference on Autonomous Agents and Multiagent Systems*. 692–700.

[7] Bin Li, Dong Hao, Hui Gao, and Dengji Zhao. 2022. Diffusion auction design. *Artificial Intelligence* 303 (2022), 103631.

[8] Bin Li, Dong Hao, and Dengji Zhao. 2020. Incentive-compatible diffusion auctions. *arXiv preprint arXiv:2001.06975* (2020).

[9] Bin Li, Dong Hao, Dengji Zhao, and Makoto Yokoo. 2019. Diffusion and Auction on Graphs. In *Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI 2019*. 435–441.

[10] Bin Li, Dong Hao, Dengji Zhao, and Tao Zhou. 2017. Mechanism design in social networks. In *Thirty-First AAAI Conference on Artificial Intelligence*.

[11] Haolin Liu, Xinyuan Lian, and Dengji Zhao. 2022. Diffusion multi-unit auctions with diminishing marginal utility buyers. *arXiv preprint arXiv:2201.08616* (2022).

[12] Xiang Liu, Weiwei Wu, Minming Li, and Wanyuan Wang. 2021. Budget Feasible Mechanisms Over Graphs. In *Thirty-Fifth AAAI Conference on Artificial Intelligence, AAAI 2021*. 5549–5556.

[13] Ahmed Moustafa, Pankaj Mishra, and Nagoya Kogyo Daigaku. 2021. A Diffusion Mechanism for Multi-Unit Commodity Allocation in Economic Networks. *Electronic Commerce Research and Applications* (2021), 101078.

[14] Seiji Takanashi, Takehiro Kawasaki, Taiki Todo, and Makoto Yokoo. 2019. Efficiency in truthful auctions via a social network. *arXiv preprint arXiv:1904.12422* (2019).

[15] William Vickrey. 1961. Counterspeculation, auctions, and competitive sealed tenders. *The Journal of finance* 16, 1 (1961), 8–37.

[16] Bo You, Ludwig Dierks, Taiki Todo, Minming Li, and Makoto Yokoo. 2022. Strategy-Proof House Allocation with Existing Tenants over Social Networks. In *Proceedings of the 21st International Conference on Autonomous Agents and Multiagent Systems*. 1446–1454.

[17] Wen Zhang, Dengji Zhao, and Hanyu Chen. 2020. Redistribution Mechanism on Networks. In *Proceedings of the 19th International Conference on Autonomous Agents and Multiagent Systems, AAMAS '20, Auckland, New Zealand, May 9-13, 2020*. International Foundation for Autonomous Agents and Multiagent Systems, 1620–1628.

[18] Wen Zhang, Dengji Zhao, and Yao Zhang. 2020. Incentivize Diffusion with Fair Rewards. In *ECAI 2020 - 24th European Conference on Artificial Intelligence (Frontiers in Artificial Intelligence and Applications, Vol. 325)*. 251–258.

[19] Yao Zhang and Dengji Zhao. 2022. Incentives to Invite Others to Form Larger Coalitions. In *Proceedings of the 21st International Conference on Autonomous Agents and Multiagent Systems*. 1509–1517.

[20] Dengji Zhao. 2021. Mechanism Design Powered by Social Interactions. In *Proceedings of the 20th International Conference on Autonomous Agents and MultiAgent Systems*. 63–67.

[21] Dengji Zhao. 2022. Mechanism design powered by social interactions: a call to arms. In *Proceedings of the Thirty-First International Joint Conference on Artificial Intelligence, IJCAI-22*. 5831–5835.

[22] Dengji Zhao, Bin Li, Junping Xu, Dong Hao, and Nicholas R. Jennings. 2018. Selling Multiple Items via Social Networks. In *Proceedings of the 17th International Conference on Autonomous Agents and MultiAgent Systems, AAMAS 2018*. 68–76.