How Does Fairness Affect the Complexity of Gerrymandering?

Extended Abstract

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ABSTRACT

Gerrymandering is a common way to externally manipulate district-based elections where the electorate is (artificially) redistricted with an aim to favour a particular political party to win more districts in the election. Formally, given a set of m possible locations of ballot boxes and a set of n voters (with known preferences) is it possible to choose k specific locations for the ballot boxes so that the desired candidate wins in at least l of them? Lewenberg et al. [AAMAS '17] and Eiben et al. [AAAI '20] studied the classical and fine-grained complexity (respectively) of the gerrymandering problem.

In recent years, the research direction of studying the algorithmic implications of introducing fairness in computational social choice has been quite active. Motivated by this, we define two natural fairness conditions for the gerrymandering problem and design a near-optimal algorithm. Our two new conditions introduce an element of fairness in the election process by ensuring that:

- the number of voters at each ballot box is not unbounded, i.e., lies in the interval $[\mathtt{lower}, \mathtt{upper}]$ for some given parameters $\mathtt{lower}, \mathtt{upper}$
- the margin of victory at each ballot box is not unbounded, i.e., lies in the interval $[\mathtt{margin}_{\mathtt{low}}, \mathtt{margin}_{\mathtt{up}}]$ for some given parameters $\mathtt{margin}_{\mathtt{low}}, \mathtt{margin}_{\mathtt{up}}$

For the real-life implementation of redistricting, i.e., when voters are located in $\mathbb{R}^2$, we obtain the following upper and lower bounds for this fair version of the gerrymandering problem:

- There is an algorithm running in $(m+n)^{O(\sqrt{k})} \cdot |C|^{(\mathtt{upper}+\mathtt{lower}+\mathtt{margin}_{\mathtt{up}}+\mathtt{margin}_{\mathtt{low}})}$ time where C is the set of candidates participating in the election.
- Under the Exponential Time Hypothesis (ETH), we obtain an *almost tight* lower bound by ruling out algorithms running in $f(k, n, \mathtt{upper}, \mathtt{lower}) \cdot m^{o(\sqrt{k})}$ time where f is any computable function. The lower bound holds even when $\mathtt{margin}_{\mathtt{low}} = 1 = \mathtt{margin}_{\mathtt{up}}$, $k = l$ and there are only 2 candidates.

KEYWORDS

Fairness, Gerrymandering, Computational social choice, Elections, Manipulation, Parametrized complexity, Exponential Time Hypothesis

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1 INTRODUCTION

Elections are a fundamental process in our lives: a group of agents vote according to their individual preferences to select a final outcome from a given set of outcomes. Given the high stakes, it is highly important to preserve the sanctity of an election from manipulation by either internal or external sources. Seminal results [15, 24] showed that most standard voting rules are susceptible to (internal) manipulation: the outcome of the election can be changed significantly even if one agent votes differently from their true preference! To add to the bad news, there is evidence [9] from economics and political science that many of the voting systems used in real life actually incentivise voters to deviate from their true preferences.

A series of highly-influential papers [1–3] initiated the study of manipulation in various different voting scenarios from the viewpoint of computational complexity: given that (internal) manipulation is possible [15, 24] how easy or hard is to actually achieve a specific outcome in a given voting system? We refer the interested reader to [5, 12–14] for more information about this active area of research in computational social choice.

Internal manipulation is typical of the form where a coalition of voters strategically votes (often different from their true preferences) to ensure the victory (or loss) of a specific candidate [5][Chapter 6]. External manipulation on the other hand asks whether an agent who is not even participating in the election can still manipulate it in a way to ensure the victory (or loss) of a specific candidate. This can be achieved in various different ways: adding or removing voters or candidates [5][Chapter 7.3], bribing voters to change their preferences [5][Chapter 7.4], redistricting in district-based elections [11, 18], etc. In this paper, we focus on election manipulation by redistricting in district-based elections. An axiomatic study of Gerrymandering was introduced in [23]. A notion of fairness in the context of gerrymandering was introduced in [22]. Their fairness criteria ensure the proportion of voters remains the same after redistricting.

1.1 Our Model: Fairness to combat Gerrymandering

In this paper, we impose the following two fairness rules with a view towards preserving the sanctity of the voting mechanism:

- The number of voters at each of the ballot boxes is bounded i.e. $\leq \mathtt{upper}$ and $\geq \mathtt{lower}$.
- The margin on victory at each of the ballot boxes is also bounded i.e. $\leq \mathtt{margin}_{\mathtt{up}}$ and $\geq \mathtt{margin}_{\mathtt{low}}$.

We bound the number of voters at each of the ballot boxes in order to avoid the demography corresponding to each of the ballot boxes being skewed. On the other hand, the margin of victory is an important parameter to ensure fairness in voting. It is often noticed that the prediction of the exit poll varies widely with the real outcome of the voting. One of the main reasons behind this is the occurrence of different malpractices like bribery and rampant rigging at the time of voting. The "margin of victory" parameter inhibits these malpractices to some extent by increasing the chances of recounting or in some cases re-polling if it is found after the election result that the margin of victory exceeds the boundaries. This parameter is also being used to measure the number of votes that would need to change with an aim to alter a parliamentary outcome for single-member preferential electorates. Various earlier works [4, 8, 20, 25] showed how to compute the margin of victory for different voting rules and draw its impact in the real scenario.

Here we study the impact of the above fairness rules on gerrymandering. Additionally, we also impose the standard condition that a voter cast their vote at the ballot box located nearest to her. More specifically, we study the following problem (we call it "Fair-Gerrymandering") defined as follows.

Fair-Gerrymandering-$(\mathcal{X}, \rho)$
Input: A set of candidates C, a set $\mathcal{V}$ of n voters located at points in $\mathcal{X}$ whose preferences are known, a set $\mathcal{B}$ of m possible ballot box locations in $\mathcal{X}$ and a specific candidate "OUR" $\in C$
Parameters: k, l, m, n, $\texttt{upper}$, $\texttt{lower}$, $\texttt{margin}_{\texttt{low}}$, $\texttt{margin}_{\texttt{up}}$
Assumptions:

- Each voter votes at the ballot box nearest to them, where distances are calculated using the metric ρ.
- The *plurality* rule is used: each voter votes for their top-ranked candidate, and a ballot box is won by the candidate who secures the most votes. No ties.
- The number of voters voting for a candidate at every ballot box is bounded i.e., $\leq \texttt{upper}$ and $\geq \texttt{lower}$.
- The margin of victory at every ballot box is $\leq \texttt{margin}_{\texttt{up}}$ and $\geq \texttt{margin}_{\texttt{low}}$.

Question: Is there a set $\mathcal{P} \subseteq \mathcal{B}$ such that $k = |\mathcal{P}|$ such that opening ballot boxes at locations in $\mathcal{P}$ then "OUR" candidate wins at least l of the ballot boxes for some $l \leq k \leq m$.

1.2 Our Results

In this paper, we initiate the study of the algorithmic complexity of the Fair-Gerrymandering problem. On the algorithmic side, we obtain the following result:

Theorem 1. *If C is the set of candidates in an election, n is the number of voters and m is the number of possible ballot box locations in the plane, then Fair-Gerrymandering-$(\mathbb{R}^2, \ell_2)$ is solvable in time $(m+n)^{O(\sqrt{k})} \cdot |C|^{(\texttt{upper}+\texttt{lower}+\texttt{margin}_{\texttt{up}}+\texttt{margin}_{\texttt{low}})}$ where k is the number of ballot boxes for the election.*

A brute-force search for a solution will run in $m^k n^{O(1)}$. Clearly, our algorithm is more efficient than an exhaustive search. We complement this algorithm with an almost-matching lower bound:

Theorem 2. *For any $d \geq 2$, under the Exponential Time Hypothesis (ETH), the Fair-Gerrymandering-$(\mathbb{R}^d, \rho)$ problem cannot be solved in $f(k, n, \texttt{upper}, \texttt{lower}) \cdot m^{o(k^{1-1/d})}$ time where f is any computable function, n is the number of voters, and k is the number of the ballot boxes opened, m is the total number of possible locations of ballot boxes and ρ is either the ℓ_∞-metric or the ℓ_q-metric for some $q \geq 1$. This lower bound holds even when there are only 2 candidates, $k = l$ and $\texttt{margin}_{\texttt{low}} = 1 = \texttt{margin}_{\texttt{up}}$.*

Recall that the Exponential Time Hypothesis (ETH) is a standard assumption [19] in parameterized complexity theory which states that the 3-SAT problem cannot be solved in $2^{o(N)}$ time where N is the number of variables [16, 17].

Note that since $1 \leq \texttt{lower} \leq \texttt{upper} \leq n$, the terms $\texttt{lower}$ and $\texttt{upper}$ are redundant in the first term of the claimed lower bound for the running time in Theorem 2. However, we have chosen to include them here for the sake of completeness so that the involvement of each of the four fairness parameters ($\texttt{lower}$, $\texttt{upper}$, $\texttt{margin}_{\texttt{low}}$, $\texttt{margin}_{\texttt{up}}$) in Theorem 2 is explicitly clear.

Comparison of our results & techniques to [10]:
Eiben et al. [10] studied the "vanilla" version, i.e., without any fairness constraints, of the Gerrymandering-$(\mathbb{R}^2, \ell_2)$ problem. Note that this "vanilla" version of the Gerrymandering problem, i.e., the Gerrymandering-$(\mathcal{X}, \rho)$ problem studied in [10], is a special case of the Fair-Gerrymandering-$(\mathcal{X}, \rho)$ problem with the following "extreme" values of some of the parameters:

- $\texttt{margin}_{\texttt{low}} = 1$ and $\texttt{margin}_{\texttt{up}} = n$
- $\texttt{lower} = 0$ and $\texttt{upper} = n$

Eiben et al. [10] designed an $(m+n)^{O(\sqrt{k})}$ algorithm along with a lower bound of $f(k, n) \cdot m^{o(\sqrt{k})}$ under ETH. We now briefly compare our results & techniques to those of Eiben et al. [10]:

Algorithmic result. The key idea of our algorithm lies in using the well-known separator theorem of Voronoi diagrams by Marx and Pilipczuk [21] in a recursive way. The non-trivial part of the technique comes from the efficient handling of partial solutions. At each step of the recursion, we combine partial solutions from the lower level. As we don't know how the final solution will look like we may cut a district several times into smaller pieces during the recursion. We maintain possible solutions for all such pieces to compute the final district partitioning, ensuring the fairness criteria.

Lower bound. Our reduction is similar to that of [10] for the "vanilla" version of Gerrymandering, but reducing from the $(k \times k)$-Grid-Tiling-$\geq$ version of the problem (instead of $(k \times k)$-Grid-Tiling as done in [10]) helps us to simplify some of the arguments. Further, we are able to generalise our reduction which works well for any metric ℓ_q^d where $1 \leq q \leq \infty$ and d are arbitrary. This reduction is from the d-dimensional $\geq$-CSP problem which has been recently used to show lower bounds for various problems in computational geometry [6, 7]. Note that if we set $k = l$ and $\texttt{margin}_{\texttt{low}} = 1 = \texttt{margin}_{\texttt{up}}$ in the reduction, it implies that our desired candidate has to win each ballot box by exactly one vote. For the other two parameters, we only need to use the naive bounds $1 \leq \texttt{lower} \leq \texttt{upper} \leq n$.

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