Bipartite Matching for Repeated Allocation Problems

Doctoral Consortium

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ABSTRACT

Many applications involving the allocation of resources or tasks can be modeled as matching problems in bipartite graphs. In many of these applications, allocation is performed multiple times. An example is the allocation of classrooms to course instructors, which is done every semester. To improve their chances of being assigned, instructors may relax some of their restrictions. Another example is course and classroom assignments made for weekly workdays. In this case, however, the assignment is made multiple times at once (once for each workday of the week). Finally, in task assignment problems where resources are reusable, each resource can be assigned multiple times. We describe algorithmic solutions to some of these problems and demonstrate their effectiveness in applications such as car teleoperation, desk sharing, and classroom assignment. Finally, we discuss several directions and ideas for extending our work and solving other relevant problems.

KEYWORDS

Bipartite matching; Resource allocation; Teleoperation

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1 INTRODUCTION

There are many practical contexts in which a set of **agents** must be suitably matched with a set of **resources**. Examples of such contexts include matching classes with classrooms [8], medical students with hospitals [10], hot desking or shared workspaces [2, 15], etc. In some contexts, the matching is done periodically, such as when students are assigned to hospitals, while in other contexts, it should be done multiple times in parallel, such as when desks are shared, which must be done for a whole week in total. Similarly, in some contexts, such as assigning papers to reviewers [7] or assigning customers to taxis [5, 6] **tasks** should be matched with **workers**, and this may happen more than once. In these contexts, the tasks are comparable to agents, and the workers are the resources. In some of these problems, the agents and resources are known in advance, while in other cases, one or both arrive dynamically. When they arrive dynamically, it might be a good idea to repeat the matching periodically so that, on the one hand, the tasks or agents do not have to wait too long for resources. On the other hand, an appropriate matching is performed. In the following sections, we describe our results about the contexts mentioned above and point out some directions for future work. A summary of the problems considered in each paper is in Table 1

Papers	# of Matchings	Relaxable Constraints	Offline/ Online	Delayed Assignments
[12],[13]	One	Yes	Offline	No
[14]	Multiple	Sometimes	Offline	No
[1]	Repeated, one at a time	No	Online	Yes

Table 1: Problems considered in each paper.

2 MAXIMIZING ALLOCATION LIKELIHOOD WITH MINIMUM REGRET

Many scenarios where agents with restrictions compete for resources can be cast as maximum matching problems on bipartite graphs. We focus on resource allocation problems where agents may have restrictions that make them incompatible with some resources. We assume that a Principal chooses a maximum matching randomly so that each agent is matched to a resource with some probability. Agents would like to improve their chances of being matched by modifying their restrictions. Improving the chances could be important if the matching is to be repeated in the near future. The Principal's goal is to advise an unsatisfied agent to relax its restrictions so that the total cost of relaxation is within a budget and the increase in the probability of being assigned a resource is maximized.

In our work (Trabelsi et al. [12, 13]) we present the following results: 1. The matching advice problem. We develop a formal framework for advising agents in a resource allocation setting viewed as a matching problem on an agent-resource bipartite graph. We formulate a budget-constrained optimization problem to generate suitable relaxations of an unmatched agent's restrictions so as to maximally increase the probability that the agent will be matched. We identify and study different forms of restrictions arising from agent restrictions and resource properties in real-world applications.
2. Complexity of improving the likelihood of matching. We show that, in general, the budget-constrained optimization problem is **NP**-hard.
3. Algorithms for improving the likelihood of matching. Under uniform costs for relaxing restrictions and uniform random selection of maximum matchings, we present algorithmic results for some classes of restrictions. Specifically, we present an efficient approximation algorithm (with a performance guarantee of $(1 - 1/e)$) for the Multi-Choice Single-Restriction case. This result relies on the

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Problem	Requirements
MRM/MaxSA-MRM	Find a k(**M**ulti)-**R**ound **M**atching that satisfies all the agents/maximum number of agents.
MaxTB-MRM	Find a k-round matching that maximizes a valid benefit function.
AG-MRM/ AG-MaxSA-MRM	Find feasible sets of relaxations for the agents and a k-round matching in the resulting compatibility graph such that all agents/maximum number of agents are satisfied.

Table 2: Overview of problems.

submodularity of the objective function. For another class called *threshold-like* restrictions, we develop a fixed parameter tractable algorithm, assuming that the budget and the cost of removing each restriction are non-negative integers.

4. Experimental Study. We study the performance of our recommendation algorithms on both synthetic datasets as well as two real-world datasets. The latter datasets arise in the context of assigning classrooms to courses and matching children with activities.

3 RESOURCE SHARING THROUGH MULTI-ROUND MATCHINGS

Applications such as employees sharing office spaces over a work-week can be modeled as problems where agents are matched to resources over multiple rounds. Agents' requirements limit the set of compatible resources and the rounds in which they want to be matched. Viewing such an application as a multi-round matching problem on a bipartite compatibility graph between agents and resources, we show that a solution (i.e., a set of matchings, with one matching per round) can be found efficiently if one exists, and propose extensions for the case when a solution does not exist. The various problems considered in this work are in Table 2. Our results (Trabelsi et al. [14]) are below.

(a) An efficient algorithm for MaxTB-MRM. For a general class of benefit functions that satisfy certain properties, we show that the MaxTB-MRM problem can be solved efficiently by a reduction to the maximum weighted matching problem. A simple example of such a function (where each agent receives a benefit of 1 for each round in which it is matched) represents a **utilitarian social welfare** function [16]. Our efficient algorithm for this problem yields as a corollary, an efficient algorithm for the MRM problem. Our algorithm can also be used for a more complex benefit function that models a **Rawlsian social welfare function** [9, 11], where the goal is to maximize the minimum satisfaction ratio over the agents.

(b) Maximizing the number of satisfied agents. Given a multi-round matching, we say that an agent is **satisfied** if the matching satisfies all its requirements. The objective of finding a multi-round matching that satisfies the largest number of agents can be modeled as the problem of maximizing the total benefit by specifying a simple benefit function for each agent. However, such a benefit function *doesn't* have the diminishing returns property. We show that this optimization problem is **NP**-hard.

(c) Advice generation. We show that AG-MRM is **NP**-hard. Recall that the AG-MRM problem requires that each agent must be satisfied with the new compatibility graph (obtained by relaxing the suggested restrictions). The hardness directly implies the hardness of the problem where the advice must lead to a matching that satisfies the maximum number of agents (AG-MaxSA-MRM). We present two solution approaches for the AG-MaxSA-MRM problem: (i) an integer linear program to find an optimal solution and (ii) a pruned local search heuristic that uses our algorithm for MRM to generate solutions of good quality.

(d) Experimental results. We present a back-to-the-lab desk-sharing study that has been conducted in an AI lab at a Bar-Ilan university to facilitate lab personnel intending to return to the workplace during the COVID-19 epidemic. This study applies our algorithms to guide policies for returning to work. In addition, we present an experimental evaluation of our algorithms on several synthetic datasets, as well as on a dataset for matching courses to classrooms.

4 ONLINE MATCHING PROBLEMS WITH OFFLINE REUSABLE RESOURCES AND DELAYED ASSIGNMENTS

Many applications where tasks should be assigned to agents can be modeled as matching in bipartite graphs. Motivated by a work by Dickerson et al. [3], we consider applications where agents are static, and tasks arrive dynamically. However, we consider a setting in which rejection of a task may have significant adverse effects on the requester, therefore, performing the task with some delay is preferred over complete rejection. The performance time of a task depends on the task, the agent, and the assignment, and only its distribution is known. The actual time is known only after the task performance when the agent is available for a new assignment. We consider such applications to be one of two arrival types. With the first type, the arrival distribution is known in advance, while with the second type there is no assumption about the arrival times and order. Our results (Ackerman Viden et al. [1]) are bellow.

(a) Known Arrival Distribution We develop a novel online algorithm with a competitive ratio of 0.5. we show that there are situations where the algorithm rejects too many tasks and therefore propose a heuristic variant of the algorithm.

(b) Unknown Arrival Distribution We adopt a proof of Gong et al. [4] and show that there is no online algorithm with a competitive ratio better than $O(\log T/T)$.

(c) Experimental Study When the distribution is known, we compared the algorithm with a competitive ratio using the proposed heuristic and with a greedy algorithm that does not use the arrival distribution. The comparison was performed on a dataset based on both simulations and real-world data. We have shown that the heuristic based on the competitive ratio algorithm performs significantly better than the greedy heuristic in most situations.

5 DIRECTIONS FOR FUTURE RESEARCH

First, there are some different settings for each of the works presented above that need to be explored further. For example, maximizing allocation likelihood could be considered in online settings where agents or resources arrive dynamically. Second, trust in allocations should be further explored. To improve trust, we could consider adding explanations to proposed allocations. Finally, we could consider making allocations in problems modeled as other graph problems, such as matchings in general graphs and network flows.

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