# Mechanism Design for Heterogeneous and Distributed Facility Location Problems

Doctoral Consortium

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## ABSTRACT

As a field of study that integrates game theory and algorithm design, algorithmic game theory aims to design efficient algorithms in environments with strategic agents. In this thesis, we investigate one of algorithmic game theory's main classes of problems, namely facility location problems. In the most classical setting, the goal is to locate one facility on a line given the reported positions of strategic agents, who aim to be as close to the facility as possible so that the agents do not benefit from reporting false information and some social objective function is (approximately) optimized. Many researchers have recently proposed extensions to this original facility location problem. We investigate some variants of the single-facility location problem, particularly the discrete heterogeneous two-facility location problem and the distribution facility location problem. For both of them, we devise deterministic (strategyproof) mechanisms with nearly tight performance guarantees.

## KEYWORDS

Algorithmic Game Theory; Mechanism design; Facility location problem

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## 1 INTRODUCTION

Algorithmic mechanism design [16] is a branch of algorithmic game theory that aims at designing strategyproof mechanisms for dealing with strategic behaviour in combinatorial optimization problems, such as scheduling, resource allocation, and facility location. Providing incentives to the participating agents to be truthful is usually done by monetary transfers, but there are also settings, like the facility location problem, where the use of money is prohibited, leading to the area of *mechanism design without money*.

Facility location problems have been studied extensively from the perspective of approximation algorithms. The main problem is determining the optimal location of facilities to minimize transport costs for the customers served (e.g., see [11]). In mechanism design, the problem was implicitly considered for the first time by Moulin [15], who considered settings with agents that have single-peaked preferences and provided a characterization of the class of strategyproof mechanisms. Since the work of [17], who used several facility location problems as examples of approximate mechanism design without money, research in this area has flourished, and many variants have been extensively studied in the literature. Facility location problems are now possibly the most widespread and well-studied problems in economics and computer science [8, 13, 17].

The classic single-facility location problem has a set of selfish agents with private locations on a line. Our task is to place a public facility (e.g. a library or a park) [14]. A mechanism is a function that takes as input the reported positions of the agents and outputs the location of the facility. Each agent wants to be as close as possible to the facility and has a cost that is equal to her distance from the facility. As the mechanism is publicly known, an agent may lie about her position if doing so decreases her cost. To avoid this, we want the decision of the mechanism to be done such that no agent can benefit from misreporting no matter what other agents do. Typically, we also want to optimize an objective function, such as to minimize the *social cost* (the sum of costs for all agents) or the *maximum cost* (the maximum cost for all agents). The performance of the mechanism is evaluated by its *approximation ratio*, which is the worst-case ratio between the objective value of the solution computed by the mechanism and the minimum possible objective value of all feasible solutions (in all possible instances). So, our aim is to design strategyproof mechanisms with the lowest possible approximation ratio (close to 1).

The recent survey [4] provides a comprehensive introduction to the different facility location problems that have been considered over the years beyond the single-facility location problem discussed above. This thesis investigates two types of facility location problems: The discrete truthful heterogeneous two-facility location problem and the continuously distributed facility location problem.

Besides facility location problems, mechanism design without money has been successfully applied in a wide range of other problems and fields, such as healthcare [1] and clustering [10], and even problems that are not geographical, such as finding the best router location on a network [9], or electing a candidate to represent people with different political views [5]. These problems involve one or more selfish agents who are asked to report their private information as part of the input.

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## 2 CONTRIBUTIONS

The thesis focuses on designing strategyproof mechanisms for facility location problems with good performance. We have focused on the following two problems.

### Discrete Truthful Heterogeneous Two-Facility Location problem

The present study considers a scenario in which agents occupy nodes of a discrete line graph, where the agents' positions are assumed to be common knowledge. They have approval preferences over two facilities that can be placed at different nodes of the line. The cost of an agent is defined as the total distance from the facilities she approves, and the goal is to determine the locations of the two facilities in a way that both incentivizes the agents to report their preferences truthfully and achieves a good approximation of the minimum total (social) cost or the maximum cost among all agents.

Our work [12] extends the prior research of Serafino and Ventre [18], who provided bounds on the approximation ratio of deterministic and randomized mechanisms in terms of the social cost and the maximum cost. However, this study focuses exclusively on deterministic mechanisms and improves upon the bounds of Serafino and Ventre [18] for both objectives when each agent approves at least one facility.

Regarding the social cost, this study shows a lower bound of $4/3$ on the approximation ratio of any deterministic mechanism and designs a strategyproof mechanism with an approximation ratio of at most $17/4 = 4.25$. These results improve upon the corresponding bounds of $\frac{9}{8}$ and $n-1$, where $n$ is the number of agents, shown by Serafino and Ventre [18]. Additionally, this study designs a mechanism that achieves the best-possible bound of $\frac{4}{3}$ for instances with three agents.

Regarding the maximum cost objective, Serafino and Ventre [18] showed that the best possible approximation ratio of deterministic mechanisms is between $\frac{3}{2}$ and 3. However, this study shows a tight lower bound of 2 on the approximation ratio of all strategyproof mechanisms.

### Distributed Facility Location problem

For this research direction, we address the facility located on a line of real numbers that is partitioned into disjoint symmetric districts of the same size, where agents have positions on the line. The objective is to optimize a given cost function or to prevent strategic behaviour. A mechanism works in two steps: For each district, it chooses a point that is representative of the positions reported by the agents in the district and then decides on one of these representative points as the final location. Following the work of [3], in our work [6], we focus on the following cost objectives: The total distance of the agents (*social cost*); the maximum distance among all agents (*max cost*); the total maximum agent distance in each district (*Sum-of-Max cost*); the maximum total agent distance in each district (*Max-of-Sum cost*). We consider two classes of mechanisms: *Unrestricted mechanisms*, which assume that the agents directly provide their true positions as input, and *strategyproof* mechanisms, which deal with strategic agents and aim to incentivize them to report their positions truthfully.

Our first contribution is the design of a novel unrestricted mechanism that achieves an approximation ratio (called *distortion* in our paper, following the distortion literature [2]) of 2 for the social cost; this settled a question left open in the work of [7] in the affirmative, matching their lower bound of 2. Moreover, we provide unrestricted mechanisms and lower bounds for the remaining objectives to establish that these mechanisms achieve the best possible distortion. Interestingly, and perhaps unexpectedly, our mechanism for the Sum-of-Max objective is optimal; that is, it achieves a distortion of 1. This demonstrates that for this particular objective, the distributed nature of the decision-making does not influence the quality of the decision at all and stands in contrast to the results of [3] for the same objective in the discrete setting.

Next, we consider strategyproof mechanisms, i.e., mechanisms that do not incentivize the agents to misreport their locations. This type of mechanism was considered by [7], who settled their distortion for the social cost, which is 3. Strategyproof mechanisms were not previously studied for the remaining three objectives, not even in the discrete setting of [3]. We show tight bounds by carefully composing centralized statistic mechanisms for choosing the district representatives and the final location. Our bounds are 3 for the max cost and $1+\sqrt{2}$ for the Sum-of-Max and Max-of-Sum objectives.

## 3 FUTURE WORK

Many extensions could be considered beyond the particular models studied in the first project. One could study settings with more than just two facilities, settings where the positions of the agents are their private information and can report empty nodes as their positions, settings with different heterogeneous preferences such as fractional(small part in amount) or obnoxious(unwanted) ones, and also settings with more general location graphs such as trees or regular graphs.

For the second project, there are several interesting future directions, such as extending our work to more general metric spaces, defining meaningful objectives, and studying similar questions about efficiency and strategyproofness. Beyond the single-facility location problem we studied, one could consider settings with more facilities and agents with heterogeneous preferences over the facilities.

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