Altruism, Collectivism and Egalitarianism: On a Variety of Prosocial Behaviors in Binary Networked Public Goods Games

Jichen Li
CFCS, School of Computing
Peking University
Beijing, China
limo923@pku.edu.cn

Xiaotie Deng*
CFCS, School of Computing
CMAR Institute for AI
Peking University
Beijing, China
xiaotie@pku.edu.cn

YuKun Cheng*
School of Business
Jiangnan University
Wuxi, China
ykcheng@amss.ac.cn

Yuqi Pan
Peking University
Beijing, China
pyq0419@stu.pku.edu.cn

Xuanzhi Xia
Peking University
Beijing, China
2000012514@stu.pku.edu.cn

Zongjun Yang
Peking University
Beijing, China
allenyzj@stu.pku.edu.cn

Jan Xie
Cryptape
Hangzhou, China
jan@cryptape.com

ABSTRACT

Binary Networked public goods (BNPG) game consists of a network $G = (V, E)$ with n players residing as nodes in a network and making a YES/NO decision to invest a public project. Examples of such public projects include face mask wearing during a pandemic, crime reporting and vaccination, etc. Most of the conventional modes of BNPG games solely posit egoism as the motivation of players: they only care about their own benefits. However, a series of real-world examples show that people have a wide range of prosocial behaviors in making decisions. To address this property, we introduce a novel extension of BNPG games to account for three kinds of prosocial motivations: altruism, collectivism, and egalitarianism. We revise utility functions to reflect different prosocial motivations with respect to the welfare of others, mediated by a prosocial graph.

We develop computational complexity results to decide the existence of pure strategy Nash equilibrium in these models, for cases where the prosocial graph is a tree, a clique or a general network. We further discuss the Prosocial Network Modification (PNM) problem, in which a principal can change the network structure within a budget constraint, to induce a given strategy profile with respect to an equilibrium. For all three types of PNM problems, we completely characterize their corresponding computational complexity results.

KEYWORDS

Public Goods Game; Network Game; Equilibrium Computation; Prosocial Behaviors

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* Corresponding author.

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1 INTRODUCTION

Public goods are non-excludable and non-rivalrous goods that benefit all community members. Each individual can decide whether or not to invest in a public good; several others will benefit from the investment. Public goods broadly exist in various social scenarios, including vaccination, open-source software, and a clean community environment. However, these public goods are often under-invested from a societal standpoint: vaccination rate remains low, source codes are rarely shared, and many public areas are badly polluted. In these circumstances, self-interested individuals cannot be motivated by the benefits they bring to others. One of the fundamental reasons for such failure is that though individuals are connected in the sense that one's investment benefits others, they are not "socially connected", in the sense that one doesn't experience the gain or loss she causes to others. In economic terms, individual decisions cause externality, which is not internalized.

Public goods' investment in a community varies significantly because of social factors, including social bonds, norms, and values [16, 28]. The greater sense of belonging people have to the community, the more likely they are to invest in their relationships. Similarly, more efforts will be spent on public affairs if individuals are more willing to share, cooperate, and take responsibility. Such intents to benefit others in socially accepted ways are encompassed by the concept of *prosociality*. In social networks, prosociality involves various types of beliefs and behaviors, including altruism, collectivism, egalitarianism, and so on [3, 49]. Different types of prosociality may internalize individual externality in different ways. Promoting prosociality in social networks is thus an important approach to encouraging individual investment in public welfare. For example, advertisements on vaccination and mask-wearing aim to enhance people's sense of responsibility, and reports on disadvantaged groups may raise people's concern for charity and fairness.

The utilization of prosociality in social networks raises a series of questions. First, what are the possible types of agent prosociality, and their respective types of influence on individual actions? Further, can prosociality be utilized to promote investment on public good? The algorithmic feasibility of *designing* or *modifying* prosociality to induce certain outcomes in a social network requires in-depth discussion. These are the general social network problems we are concerned about in this paper.

Prosociality and its design can be a meaningful question in many strategic settings. We focus on networked public good games with binary decisions, namely binary networked public good (BNPG) games. Each player decides to invest in public goods or not, and receives the reward from the public goods invested by herself and her neighbors. Public good games well models the scenarios in which some kinds of public goods, such as herd immunity from vaccination, may not contribute to all people in society, but only benefits one's neighbors. A player's utility is a non-decreasing function of the public goods, minus the cost of her own investment. The binary variation further captures the social situations where all players have to make binary decisions about whether or not to invest in a public good, such as wearing a mask or not.

In this paper, we characterize three types of prosociality: altruism, collectivism, and egalitarianism, which are different ways to internalize a player's externality, i.e., reforming the player's utility function by introducing neighborhood-related items. Under different prosocial types, we provide algorithmic results for both problems of computing an equilibrium and modifying prosociality to induce a certain equilibrium. First, computing a pure strategy Nash equilibrium (PSNE) in any game is a fundamental question, as it helps social principles predict how players will act. Although it is known that deciding the existence of PSNE in a BNPG game (with altruism) is NP-Complete[47], we study the existence of PSNE problem in BNPG games with other types of prosociality. We also consider the problem on networks with special structures, including trees and cliques, which are of both theoretical and practical interest. Corresponding PSNE computational complexity results are shown in Table 1.

Table 1: PSNE results (our results are in bold).

	general	tree	clique
Egalitarianism	**NPC**	**P**	**P**
Altruism	NPC [48]	P [35]	P [35]
Collectivism	**PLS**	P [35]	P [35]

Next, we study Prosociality Network Modification (PNM) problem in the BNPG game, where edges of the prosocial network can be added or deleted subject to a cost of budget. The aim of the modification is to induce a target strategy profile, in which all players invest or social welfare is maximal, to be a PSNE. We consider both setting where prosocial network has undirected edges (corresp. symmetric prosociality) and directed edges (corresp. asymmetric prosociality). Corresponding results of PNM problem are shown in Table 2. Our study on this problem theoretically supports policymakers to strategically design social campaigns to encourage people to invest in public good. Our discussions on PNM cover both symmetric and asymmetric prosociality, and different network structures, including tree, clique, and general networks.

Table 2: PNM results (our results are in bold).

		Symmetric		Asymmetric	
		ALL	OPT	ALL	OPT
Egalitarianism	General	**NP-hard**	**NP-hard**	**P**	**P**
	Tree	**P**	**P**	**P**	**P**
	Clique	**NP-hard**	**NP-hard**	**P**	**P**
Altruism	General	NPC [47]	**NP-hard**	NPC [47]	**NP-hard**
	Tree	NPC [35]	**NP-hard**	NPC [35]	**NP-hard**
	Clique	NPC [47]	**NP-hard**	NPC [47]	**NP-hard**
Collectivism	General	**P**	**P**	**P**	**P**

2 RELATED WORK

2.1 Prosocial Behaviors in Social Networks

In general, motivations for individual prosocial behaviors are diverse, although none of them can be identified as a dominant factor [4]. For its influence on people's decisions in public good games, Kagel et al. [28] presented a framework of analysis by experiments. It further discussed different ways of modeling social factors and motivations. In addition, several pieces of literature modeled and discussed the process of individual trade-offs between social responsibility and personal utility in various ways [10, 23, 24].

One classical type of prosociality is altruism. Kagel et al. [28], Levine [33] used experimental methods to analyze the effect of altruism on public good games and discussed possible models to capture altruism in public good games. Fehr and Fischbacher [16] described how altruism influences the interaction between individuals in repeated games by experiments. Meier et al. [38] discussed the efficiency of equilibrium with respect to altruism by analyzing measure metrics like the price of anarchy. It was observed that people make different decisions in public goods games because of factors beyond self-regarding preference like incentive schemes and others-regarding preference [15, 17].

Collectivism is a special type of prosociality, the ultimate goal of which is to benefit a particular group as a whole. Dawes et al. [13] put it succinctly: "Not me or thee but we". Many studies have explored the differences and connections between collectivism and individualism. Dawes et al. [14] gave direct evidence from experiments to prove that collectivism is independent of individualism, illustrating the rich research value of collectivism. Brewer and Chen [9] pointed out that individualists are no less collectivistic than collectivists. Therefore, the concept of collectivism is culturally universal despite cultural differences among societies. Hui and Triandis [25] also provided a review of the main findings concerning the relationship between personality and cultural syndromes of individualism and collectivism.

Egalitarianism, built from the concept of social fairness, is characterized by the idea that people are equal in fundamental values. In social sciences, a broad range of studies with experiments [22, 26, 41, 49] identified egalitarianism as an important factor in social networks. Individuals with egalitarianism are those who promote the standards of fairness in their communities. In fairness standards, our work is related to the economic discussions on the definition of fair social states, including [20, 29]. Max-min fairness, which we adopt in our model, relates the concept of fairness with an optimal state where one cannot achieve utility gain without harming someone else whose utility is lower. This concept is widely used for studying computational issues in fairness problems [6, 39, 42].

2.2 Networked Public Good Games

Bramoullé and Kranton [7] started the study on public good games over networks. They focused on the traditional continuous investment model and assumed homogeneous concave utilities. An important variation of networked public good game treats player decisions as binary [19]. Based on the binary networked public goods (BNPG) game model, Yu et al. [48] initiated the algorithmic question of deciding the existence of pure-strategy Nash equilibrium (PSNE): Checking the existence of PSNE in the BNPG game is NP-hard in both homogeneous and heterogeneous settings. Maiti and Dey [36] discussed parameterized complexity of computing PSNE in BNPG games. A series of tractability results are provided when the network possesses certain properties, such as a tree or a clique. Public good games were also studied for directed networks [34, 40]. Our complexity results on BNPG games are also related to the literature of graphical games [30]. Networked public good games can be considered special graphical games where individual utility is a function of its neighbors' efforts. Closely related variants of graphical games were also broadly discussed in [12, 32, 37].

Another related line of research is social network modification. Some works studied how to modify social networks to influence people's opinions and achieve desired outcomes [2, 8, 11, 21, 44]. Several authors studied the effects of network modification on equilibrium outcomes [18, 45] or other optimization problems [43, 46]. Moreover, some works also applied network modification to BNPG games. Kempe et al. [31] initiated an algorithmic study of targeted network modifications of BNPG games. Unlike our work, it discussed modifying the relation graph rather than the prosocial graph. Another related work [47] considered the Altruism Network Modification (ANM) problem which discussed the possibility of adding an altruism graph to the original graph. Maiti and Dey [35] further considered parameter complexity of the ANM problem.

3 MODEL FORMULATION

This section formally introduces the definition of the BNPG games, an important variant of public goods games; and three types of BNPG games from the perspective of prosociality: altruism, collectivism, and egalitarianism. Additionally, the prosocial network modification (PNM) problem is introduced.

3.1 Binary Networked Public Goods Games

Networked public goods game is a game containing n players, defined on an undirected relation graph $G = (V, E)$, where $V = \{1, \cdots, n\}$ is the vertex set, representing the set of players. We mainly focus on the BNPG games, in which each player only has two strategies, that is, to invest in public goods or not. These two strategies are denoted by 1 or 0, respectively. Therefore, given a joint pure strategy profile $\mathbf{x} = (x_i)_{i\in V} \in \{0,1\}^{|V|}$, we use n_i to denote the number of the investing neighbors of player i in relation graph G, i.e., $n_i^{(\mathbf{x})} = |\{j \in V|(i,j) \in E|$. Once all players make decisions to invest or not, each one obtains egocentric utility, denoted by $g_i(x_i + n_i^{(\mathbf{x})})$, from the investments of herself and her neighbors, and suffers a cost c_i. The utility of the player is expressed as

$$U_i(\mathbf{x}) = g_i(x_i + n_i^{(\mathbf{x})}) - c_i \cdot x_i, \tag{1}$$

where $g_i : N \to R^+ \cup \{0\}$ is a non-decreasing function, and $c_i \in R^+ \cup \{0\}$ is the cost of investing for player i. We also denote $\Delta g_i(x) = g_i(x+1) - g(x)$. In this paper, we are more interested in the solution of pure strategy Nash equilibrium (PSNE).

Definition 1. *(Pure Strategy Nash Equilibrium (PSNE)) A pure strategy profile $\boldsymbol{x} = (x_1, ..., x_n)$ of BNPG game is a PSNE, if no player can deviate to achieve a higher utility, that is,*

$$U_i(x_i, \boldsymbol{x}_{-i}) \geq U_i(1 - x_i, \boldsymbol{x}_{-i}),$$

where $\boldsymbol{x}_{-i} = (x_j)_{j\in V\setminus\{i\}}$.

From the perspective of the whole network, maximizing the social welfare is important as well. Generally, the social welfare is defined as the difference between the sum of all players' egocentric utilities and all investment costs, that is,

$$SW(\mathbf{x}) = \sum_{i\in V} g_i(x_i + n_i^{(\mathbf{x})}) - \sum_{i\in V} c_i \cdot x_i. \tag{2}$$

Clearly, the optimal strategy profile to maximize the social welfare depends on the relation graph G.

3.2 Prosocial Motivation in BNPG Games

In a prosocial setting, players consider their as well as part of their neighbors' benefits to make a decision. A natural way to model the prosociality of participants is to let a player's utility not only come from her own egocentric utility g_i, but also from the egocentric utilities of her neighbors. To formalize the definition of neighborhood in a BNPG game with prosociality, a prosocial graph $H = (V, E')$ is additionally given, which may be undirected or directed, with its underlying graph contained in G. Specifically, if the prosocial graph H is undirected, then such a game is called to be *symmetric*; otherwise, such a game is *asymmetric*. Referring to the notations in [35], an undirected edge between i and j is denoted by $\{i, j\}$, and a directed edge from i to j is represented by (i, j). Different from the classical BNPG games, in which each player can only benefit the goods from herself and her neighbors in G, all players' utilities are formed by the egocentric utility from themselves and their neighbors restricted in the prosocial network H of BNPG games with prosociality. Thus, we define the restricted neighborhood of player i as $N_i = \{j \in V|\{i, j\} \in E'\}$, if H is undirected; otherwise, $N_i = \{j \in V|(j, i) \in E'\}$, i.e., the out-neighbors of i.

This work focus on three types of prosociality: altruism, collectivism and egalitarianism, which are detailedly introduced in the following by distinguishing the utility function of players.

Definition 2. ***(Altruistic type)****[47] Under the altruistic type, player i's utility function, after the deduction of investment cost, is the linear combination of her and all her neighbors' egocentric utilities,*

$$U_i^{(A)}(\boldsymbol{x}) = g_i(x_i + n_i^{(\boldsymbol{x})}) + a \sum_{j\in N_i} g_j(x_j + n_j^{(\boldsymbol{x})}) - c_i \cdot x_i, \tag{3}$$

where $a \in (0, 1)$ is a constant.

Definition 3. ***(Collectivistic type)*** *For the collectivistic type, each players are concerned about all her neighbors, and thus the prosocial network is assumed to be the relation network, i.e., $H = G$. Player i's utility function, after the subtracting investment cost, is the sum of her and all her neighbors' egocentric utilities,*

$$U_i^{(C)}(\boldsymbol{x}) = g_i(x_i + n_i^{(\boldsymbol{x})}) + \sum_{j\in N_i} g_j(x_j + n_j^{(\boldsymbol{x})}) - c_i \cdot x_i. \tag{4}$$

By Definition 2 and 3, it is not hard to see that the collectivism type is a special case of the altruism type, making it has better results on the equilibrium computation issues.

Definition 4. ***(Egalitarian type)*** *For the egalitarian type, each player is concerned with her neighbor, who has minimal egocentric utility. Thus each player i's utility function, after the deduction of investment cost, is the linear combination of her and her neighbor's with the minimal egocentric utility,*

$$U_i^{(F)}(\boldsymbol{x}) = g_i(x_i + n_i^{(\boldsymbol{x})}) + a \min_{j\in N_i} g_j(x_j + n_j^{(\boldsymbol{x})}) - c_i \cdot x_i, \tag{5}$$

where $a \in (0, 1)$ is a constant.

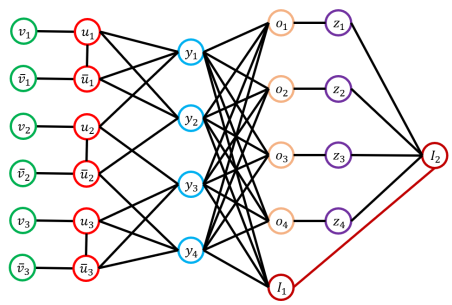

Figure 1: Example (3,B2)-SAT Graph in Theorem 1.

By this definition we assume egalitarian players are promoters of max-min fairness standards in their respective neighbourhoods. We emphasize that, although both egocentric and prosociality terms are included in individuals' utility functions, the social welfare is still defined as the sum of egocentric utilities of the players subtract the total investment.

3.3 Prosocial Network Modification

In public goods games, a major challenge is that the equilibrium may be far from desirable, because of players' "free-riding" [1] behaviors. Therefore, one of the reasonable methods is to induce a target strategy profile $\mathbf{x}^*$, for example a profile in which all players invest or a profile maximizing the social welfare, to a PSNE. Thus a principal can take a series of operations, such as community meetings, introductions and advertising. Under these operations, the participants are enforced to think more about others when they are making decisions, and thus the target profile $\mathbf{x}^*$ becomes a PSNE. We formally model this problem as the Prosocial Network Modification (PNM) problem, and an instance of PNM problem is defined as

$$(G = (V, E), H = (V, E'), (g_i)_{i \in V}, (c_i)_{i \in V}, a, C, B, \ \mathbf{x}^*).$$

In a PNM problem, a principal can modify the prosocial network H by adding or deleting edges, such that a target strategy profile is induced to be a desirable equilibrium. Specifically, if the prosocial network H is undirected, then the corresponding problem is named a *symmetric* PNM problem; otherwise, we call this problem an *asymmetric* PNM problem. Each edge $e \in E$ has an associated cost $C(e)$. If edge $e \in E'$, then $C(e)$ represents the cost to remove edge e from H; otherwise, $C(e)$ is the cost for adding edge e into H. The total cost of modification should not exceed a given budget B.

4 HARDNESS RESULTS FOR COMPUTING PSNE

The computation complexity issue for computing PSNE has been discussed for the classical BNPG games, introduced in Sec 2.1, by [48]. The authors proved that the problem of checking the existence of PSNE in BNPG games is NP-complete. For the BNPG games with asymmetric altruism, [35] showed that checking the existence of PSNE is polynomial time solvable if the input relation graph G is a tree or a clique. However, the hardness results for computing equilibrium in other types of BNPG games have not been explored yet. Thus the main task of this section is to characterize the computation complexity of deciding the existence of PSNE in BNPG games with egalitarianism and collectivism.

4.1 Computational complexity results for BNPG games with egalitarianism

For the BNPG games with egalitarianism, we prove that the problem of checking the existence of PSNE is NP-complete for any $a \in (0, 1)$, even when the prosocial graph H is the same as the relation graph G. This hardness result is obtained by reducing the problem from the NP-complete problem (3,B2)-SAT to existence of PSNE [5]. The (3,B2)-SAT is a restricted 3-SAT problem, in which each clause contains exactly three literals, and each variable occurs exactly twice positively and twice negatively in the instance.

Theorem 1. *For any $a \in (0, 1)$, deciding whether a PSNE exists in BNPG games with egalitarianism is NP-complete, even if the prosocial graph is equal to the relation graph, that is, $H = G$.*

Proof. We reduce the PSNE finding problem from the (3,B2)-SAT problem. Let $(\mathcal{L} = \{l_i : i \in [n]\}, C = \{C_j : j \in [m]\}$ be an arbitrary instance of (3,B2)-SAT problem, in which l_i represents a literal and C_j is used to denote a clause. For convenience, a function $f : \{l_i, \bar{l}_i : i \in [n]\} \longrightarrow \{u_i, \bar{u}_i : i \in [n]\}$ is defined, such that $f(l_i) = u_i$ *and* $f(\bar{l}_i) = \bar{u}_i$ for each $i \in [n]$. Then we construct an instance of BNPG game with prosocial graph $(G = (V, E), H = (V, E'), (g_i)_{i \in V}, (c_i)_{i \in V},)$ as follows:

$$\begin{aligned}
&V = \{u_i, \bar{u}_i : i \in [n]\} \cup \{v_i, \bar{v}_i : i \in [n]\} \\
&\cup \{y_j : j \in [m]\} \cup \{o_j : j \in [m]\} \cup \{z_j : j \in [m]\} \cup \{I_1, I_2\}; \\
&E' = E = \{\{u_i, \bar{u}_i\}, \{u_i, v_i\}, \{\bar{u}_i, \bar{v}_i\} : i \in [n]\} \\
&\cup \{\{y_j, f(l_1^j)\}, \{y_j, f(l_2^j)\}, \{y_j, f(l_3^j)\} : C_j = (l_1^j \vee l_2^j \vee l_3^j), j \in [m]\} \\
&\cup \{\{y_{j_1}, o_{j_2}\} : \forall j_1 \in [m] \text{ and } j_2 \in [m]\} \cup \{\{o_j, z_j\} : j \in [m]\} \\
&\cup \{\{I_1, y_j\}, \{I_2, z_j\} : j \in [m]\} \cup \{\{I_1, I_2\}\}.
\end{aligned}$$

Now let us define the utility function for the constructed BNPG game. The node $v_i, \bar{v}_i, u_i, \bar{u}_i$ correspond to the literals l_i. The egocentric utility functions of v_i and $\bar{v}_i$ are defined as follow:

$$g_{v_i}(k) = g_{\bar{v}_i}(k) = \begin{cases} 1000 & if\ k \le 1 \\ 1000k + 1000 & if\ k > 1 \end{cases}$$

The costs are $c_{v_i} = c_{\bar{v}_i} = 500$. Thus, it is not hard to deduce that v_i and u_i ($\bar{v}_i$ and $\bar{u}_i$) always adopt the same strategy in an equilibrium.

Because the degree of each u_i or $\bar{u}_i$ is 4 in the constructed BNPG game, the egocentric utility functions of all u_i ancd $\bar{u}_i$ are defined as $g_{u_i}(k) = g_{\bar{u}_i}(k) = 0$ $(if\ k \le 2)$, $g_{u_i}(3) = g_{\bar{u}_i}(3) = 3$, $g_{u_i}(4) = g_{\bar{u}_i}(4) = 5$ and $g_{u_i}(5) = g_{\bar{u}_i}(5) = 6$. The investment costs are $c_{u_i} = c_{\bar{u}_i} = 2 + 2a$. The motivation of this construction is to force $x_{u_i} + x_{\bar{u}_i} = 1$ in any equilibrium. We will prove it later.

Next we move on to constructing the utility function of nodes y_j, o_j, z_j for $j \in [m]$, which corresponds to the clause C_j. We first define $g_{y_j}(k) = 1000k + 1000m$ and $c_{y_j} = 1000 + 2a$, so that a node y_j plays the strategy of 1 in an equilibrium only when one of her neighbors u_i plays strategy of 0. For each node o_j, we define:

$$g_{o_j}(k) = \begin{cases} 10 & if\ k \le n - j \\ 2(k - n + j + 1) + 10 & if\ k > n - j \end{cases}$$

and the investment cost $c_{o_j} = 0$. Under such a construction, o_j always invests, i.e., $x_{o_j} = 1$, in an equilibrium. For node z_j, let us set $g_{z_j}(k) = 1000m$ and $c_{z_j} = a$. Under this setting, only when the investment of o_j's neighbors is large than $j-1$, node z_j plays the strategy of 1 in equilibrium.

Finally, for both I_1 and I_2, we set

$$g_{I_1}(k) = g_{I_2}(k) = \begin{cases} 55k + 20 + 20m & if\ m-k\ is\ even \\ 55k + 65 + 20m & if\ m-k\ is\ odd \end{cases}$$

and $c_{I_1} = c_{I_2} = 55 + 55a$. The purpose to introduce the pair of nodes $\{I_1, I_2\}$ is to indicate whether the (3,B2)-SAT instance can be satisfied. We will explain it later in detail.

Until now the instance construction of a BNPG game has been completed. We claim that the above constructed BNPG game has a PSNE if and only if the (3,B2)-SAT instance is a "YES" instance.

In one direction, if the (3,B2)-SAT instance is a "YES" instance, then let us consider the strategy profile $\mathbf{x}^*$ as follows:

- For $i \in [n]$, if l_i is TRUE, we set $x_{u_i} = x_{v_i} = 0\ and\ x_{\bar{u}_i} = x_{\bar{v}_i} = 1$; else, we set $x_{u_i} = x_{v_i} = 1\ and\ x_{\bar{u}_i} = x_{\bar{v}_i} = 0$.
- For each $j \in [m]$, we set $x_{y_j} = x_{z_j} = x_{o_j} = 1$.
- For I_1 and I_2, we set $x_{I_1} = 1\ and\ x_{I_2} = 0$.

Note that $g_{u_i} \ll g_{o_j} \ll g_I \ll g_{v_i} \ll g_{z_i} \ll g_{y_i}$, which helps us to distinguish the neighbor with minimal egocentric utility. We now claim $\mathbf{x}^*$ is a PSNE, by showing that none of the nodes deviate:

- For $x_{u_i} = 1$ and $x_{\bar{u}_i} = 0$, we have $n_{u_i} = 3$ and $n_{\bar{u}_i} = 2$. Because $\Delta g_{u_i}(3) + a\Delta g_{\bar{u}_i}(2) = 2 + 3a > c_{u_i} = 2 + 2a$, showing the marginal utility of u_i on investment is larger than the investment cost, u_i then would not like to deviate from strategy of 1. Similarly, since $\Delta g_{\bar{u}_i}(3) + a\Delta g_{u_i}(4) = 2 + a < c_{\bar{u}_i}$, meaning the marginal utility of $\bar{u}_i$ on investment cannot cover the investment cost, not investing is a better strategy for $\bar{u}_i$, and it has no incentive to deviate the strategy of 0. We can use the symmetric analysis for $x_{u_i} = 0$ and $x_{\bar{u}_i} = 1$.
- For each v_i and $\bar{v}_i$, as $x_{v_i} = x_{u_i}, x_{\bar{v}_i} = x_{\bar{u}_i}$, they won't deviate.
- For each y_j, because the (3, B2)-SAT instance is a "YES" instance, there must exist one TRUE literal l_i^j in clause C_j. Therefore, each y_j must have at least one neighbor literal node $f(l_i^j)$ playing strategy of 0, which we let be u_j without loss of generality. So $\Delta g_{y_j}(n_{y_j}) + a\Delta g_{u_j}(2) = 1000 + 3a > c_{y_j} = 1000 + 2a$. Therefore, y_j will not deviate.
- For each o_j, as all $x_{y_j} = 1$, $\Delta g_{o_j}(n) > c_{o_j}$, o_j will not deviate.
- For each z_j, its neighbor with minimal egocentric utility is o_j, and we have $\Delta g_{z_j}(m) + a\Delta g_{o_j}(m+1) = 2a > a$. So z_j will not deviate.
- For I_1 *and* I_2, we have $\Delta g_{I_1}(m) + a\Delta g_{I_2}(m) = 100 \times (1+a) > c_{I_1} = 55 + 55a$ and $\Delta g_{I_2}(m+1) + a\Delta g_{I_1}(m+1) = 10 \times (1+a) < c_{I_2} = 55 + 55a$, so both I_1 and I_2 have no incentive to deviate.

So, the strategy profile $\mathbf{x}^*$ is proved to be a PSNE.

In the other direction, let $\mathbf{x}^*$ be a PSNE of the BNPG instance, we claim that the corresponding (3,B2)-SAT instance is a "YES" instance. Firstly, because $c_{o_j} = 0$, strategy of 1 is the best choice for each o_j, $j \in [m]$, and thus $x^*_{o_j} = 1$. Secondly, we will prove that for all $j \in [m]$, x_{y_j} must be 1 by contradiction. Therefore, we assume some y_j nodes play the strategy of 0 and denote the number of these nodes to be $k > 0$. Based on this assumption, we have $n_{I_1} = m - k + x_{I_2}$.

(1) Observing that for each o_j, it has $m-k$ neighbors playing strategy of 1. Hence, for $j \in [1, k+1]$, $x_{z_j} = 1$, meaning $n_{I_2} = m - k + x_{I_1} + 1$.

(2) If k is odd, we prove that any strategy of I_1, I_2 can not form an equilibrium by distinguishing the following four cases.
- If $x_{I_1} = x_{I_2} = 1$, then $\Delta g_{I_2}(m-k+2) + a\Delta g_{I_1}(m-k+1) = 10 + 100a < 55 + 55a$, then I_2 will deviate from 1 to 0;
- If $x_{I_1} = 1\ and\ x_{I_2} = 0$, then $\Delta g_{I_1}(m-k) + a\Delta g_{I_2}(m-k+1) = 10 + 100a < 55 + 55a$, then I_1 will deviate from 1 to 0;
- If $x_{I_1} = 0\ and\ x_{I_2} = 0$, then $\Delta g_{I_2}(m-k+1) + a\Delta g_{I_1}(m-k) = 100 + 10a > 55 + 55a$, then I_2 will deviate from 0 to 1;
- If $x_{I_1} = 0\ and\ x_{I_2} = 1$, then $\Delta g_{I_1}(m-k+1) + a\Delta g_{I_2}(m-k+2) = 100 + 10a > 55 + 55a$, then I_1 will deviate from 0 to 1.

(3) Similarly, if k is even, we also can prove that any strategy of I_1, I_2 can not form an equilibrium by distinguishing the following four cases.
- If $x_{I_1} = 1\ and\ x_{I_2} = 1$, then $\Delta g_{I_1}(m-k+1) + a\Delta g_{I_2}(m-k+2) = 10 + 100a < 55 + 55a$, then I_1 will deviate from 1 to 0;
- If $x_{I_1} = 0\ and\ x_{I_2} = 1$, then $\Delta g_{I_2}(m-k+1) + a\Delta g_{I_1}(m-k) = 10 + 100a < 55 + 55a$, then I_2 will deviate from 1 to 0;
- If $x_{I_1} = 0\ and\ x_{I_2} = 0$, then $\Delta g_{I_1}(m-k) + a\Delta g_{I_2}(m-k+1) = 100 + 10a > 55 + 55a$, then I_1 will deviate from 0 to 1;
- If $x_{I_1} = 1\ and\ x_{I_2} = 0$, then $\Delta g_{I_2}(m-k+2) + a\Delta g_{I_1}(m-k+1) = 100 + 10a > 55 + 55a$, then I_2 will deviate from 0 to 1.

So, in strategy profile $\mathbf{x}^*$, a PSNE of the BNPG game, $x^*_{y_j} = 1$, for each y_j, $j \in [m]$. It means that at least one neighbor in the node of clause $\{f(l_1^j), f(l_2^j), f(l_3^j)\}$ playing strategy of 0. Meanwhile, we note that for each u_i and $\bar{u}_i$, the neighbor is greater than 2. On the one hand, if both x_{u_i} and $x_{\bar{u}_i}$ plays strategy of 1, then $\Delta g_{u_i}(4) + \Delta g_{\bar{u}_i}(4) = 1 + a < c_{u_i} = 2 + 2a$, u_i will deviate from 1 to 0. On the other hand, if both x_{u_i} and $x_{\bar{u}_i}$ plays strategy of 0, then $\Delta g_{u_i}(2) + \Delta g_{\bar{u}_i}(2) = 3 + 3a > c_{u_i} = 2 + 2a$, u_i will deviate from 0 to 1. Therefore, we prove that in any equilibrium, $x_{u_i} + x_{\bar{u}_i} = 1$ for each $i \in [n]$. For each $x_{u_i} = 0$, we set l_i yo be true; otherwise, l_i is set to be false. Then each clause $C_j = \{l_1^j, l_2^j, l_3^j\}$ must contain at least one literal, which is true, and thus the (3,B2)-SAT problem is a "YES" instance. It completes the proof of this claim. □

Since determining the existence of a PSNE is hard in general BPNG games with egalitarianism by Theorem 1, it is possible for us to discuss tractable special cases. Let us consider the case in which G is a tree, and prove that the problem of checking the existence of PSNE in the BNPG game with egalitarianism is polynomial time solvable if G is a tree.

THEOREM 2. *When the relation graph G is a tree, the problem of checking the existence of PSNE in BNPG game with egalitarianism can be solved in $O(d_{max}^5 \log(d_{max}) \cdot |V|)$ time, where d_{max} is the maximum degree of G.*

PROOF SKETCH. We design a bottom-up traversal algorithm to solve this problem. In our algorithm, each leaf or internal node passes a *conditional satisfiable table* to its parent. This table contains all satisfiable situations subject to its parent's strategy. For example, consider a node v and its parent u. A tuple (x_u, n_u, x_v, n_v) is in the conditional satisfiable table T_v, demonstrating that there exists an equilibrium in which the strategies of u and v are x_u and x_v, and the number of the investing neighbors of u and v are n_u and n_v, respectively. When any node's table is empty, we can conclude that the PSNE does not exist in this game. Otherwise, a PSNE exists. The full proof is presented in Appendix A. □

We then prove the existence of PSNE in BNPG game with egalitarianism is also tractable if G is a clique, with proof in Appendix B.

Theorem 3. *When the relation graph G is a clique, the problem of checking the existence of PSNE in BNPG game with egalitarianism can be solved in $O(|V|^2)$ time.*

4.2 Equilibrium computation in BNPG games with collectivism

BNPG game with collectitivsm is a special kind of the game with altruism, as the coefficient $a = 1$ and $H = G$. For this kind of BNPG game, we prove that the PSNE always exists by showing that this game is a potential game. We also prove that the PSNE finding problem is in PLS complexity [27] in this scenario. Furthermore, we propose a heuristic algorithm to compute a PSNE of BNPG game with collectivism.

Lemma 1. *BNPG games with collectivism is a potential game, whose potential function is exactly the social welfare function.*

Proof. We prove this claim based on the fact that for any player i, when i changes her strategy, only the egocentric utilities of i's neighbors are affected. So we have

$$\begin{aligned}
&SW(1,\mathbf{x}_{-i}) - SW(0,\mathbf{x}_{-i})\\
&= \sum_{j\in N_i\cup i} g_j(x_j+n_j) - \sum_{j\in N_i\cup i} g_j(x_j+n_j-1) - c_i\\
&= \{g_i(1+n_i) + \sum_{j\in N_i} g_j(x_j+n_j) - c_i\}\\
&- \{g_i(n_i) + \sum_{j\in N_i} g_j(x_j+n_j-1)\}\\
&= U_i^{(C)}(1,\mathbf{x}_{-i}) - U_i^{(C)}(0,\mathbf{x}_{-i}),
\end{aligned}$$

implying the game is a potential game. □

Theorem 4. *In BNPG game with collectivism, the PSNE maximizing the social welfare always exists.*

Proof. In a potential game, since the value of function $U_{v_i}^{(C)}$ for each player $v_i \in V$ is finite, the potential function must has a maximal value. In addition, when the potential function is maximized under a strategy profile $\mathbf{x}^*$, each player can not increase her utility by deviating her strategy, proving that $\mathbf{x}^*$ is a PSNE. □

Theorem 4 ensures the existence of PSNE. We continue to explore the complexity of the PSNE finding problem. Following theorem proves this problem is in the complexity class Polynomial Local Search (PLS) [27], which is the class of function problems that are guaranteed to have an answer, and this answer can be checked in polynomial time.

Theorem 5. *For the BNPG game with collectivism, the complexity of the PSNE finding problem is in PLS.*

Proof. As we proved BNPG game with collectivism is a potential game, we define L as a PSNE finding problem. the set of instances D_L denote PSNE finding problem in all BNPG game with collectivism. Consider the instance $I \in D_L$ with $G = (V,E), |V| = n$. A solution s for that instance is a strategy profile, that each player i playing 0 or 1. Thus, a solution consists of n bits. The set of solutions $F_L(I)$ is the set of all 2^n number of strategy profiles. The cost of each solution is the social welfare in this profile. The neighbor of a solution s is reached by flipping one player's strategy. By the definition of PSNE and potential game, when a strategy profile is an equilibrium, anyone deviating from her strategy can not increase social welfare. So that when s is a PSNE, the cost of s is locally optimal. Then the PSNE finding problem has the following properties:

- The size of a strategy profile is n bits, thus every solution $s \in F_L(I)$ is bounded and polynomial time verifiable.
- The social welfare of a strategy profile can be calculated in $O(n^2)$ time, thus the cost of all $s \in F_L(I)$ can be computed in polynomial time.
- The "neighbors" of a strategy profile $\mathbf{x}$ can be found in $O(n)$ time by flipping each player's strategy, thus for each solution s, the set of neighbors can be found in polynomial time.
- A PSNE can be verified in $O(n^2)$ time. Therefore for each s, we can check if it is locally optimal in polynomial time.
- Theorem 4 has proved that the PSNE maximizing social welfare always exists. Thus for every instance I, there exactly exists some s such that s is a locally optimal solution of I.

Therefore, we conclude that the PSNE finding problem in BNPG game with collectivism is in PLS-complexity. □

We provide Algorithm 1, i.e., the best-response dynamic algorithm, to compute a PSNE, which also can maximize the social welfare. Particularly, if all $U_{v_i}^{(C)}(k)$ are assumed to be integer-valued and bounded by poly(k), then the potential function is still polynomial. It means that the best-response dynamics eventually converges to a PSNE within polynomial iterations.

Algorithm 1: Best-Response Dynamics

```
Input: G = (V, E), U_{v_1}^{(C)}, ··· , U_{v_n}^{(C)}
Output: Nash equilibrium profile x
for i ∈ [n] do
    x_i ← 0
end
S ← 0 ;
while S = 0 do
    S ← 1 ;
    for i ∈ [n] do
        if U_{v_i}^{(C)}(1 − x_i, x_{−i}) > U_{v_i}^{(C)}(x_i, x_{−i}) then
            x_i ← 1 − x_i;
            S ← 0 ;
        end
    end
end
return x;
```

5 RESULTS FOR PROSOCIAL NETWORK MODIFICATION

This section mainly studies the computational complexity issue of PNM problem in both symmetric and asymmetric settings, with the task of computing whether a target strategy profile can be induced as a PSNE through certain modifications on the network.

5.1 Results for PNM problem with egalitarianism

For PNM problem with symmetric egalitarianism, we begin with a intractability result, showing it is NP-complete to induce a target profile in the general case. We consider the target where all players invest, and use reduction from (3, B2)-SAT.

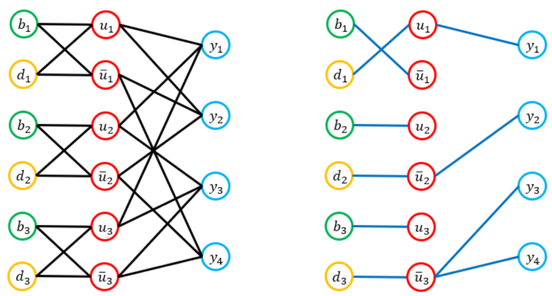

Figure 2: Example (3,B2)-SAT Graph in Theorem 6.

Theorem 6. *For the target strategy profile* $\mathbf{x}^* = (1, \cdots, 1)$*, the PNM problem with symmetric egalitarianism is NP-complete, even when the relation graph is a clique.*

Proof. To obtain this hardness result, we reduce the PNM problem with symmetric egalitarian from the (3,B2)-SAT instance ($\mathcal{L} = \{l_i : i \in [n]\}$, $C = \{C_j : j \in [m]\}$). Let us define a function $f : \{l_i, \bar{l}_i : i \in [n]\} \to \{u_i, \bar{u}_i : i \in [n]\}$, such that $f(l_i) = u_i$ and $f(\bar{l}_i) = \bar{u}_i$, $i \in [n]$. We now create an instance of PNM with symmetric egalitarian as follows. Let the initial prosocial network H be empty and $a = 1$. The relation network $G = (V, E)$ (demonstrated on the left of Fig. 2) for the input BNPG game is constructed as:

$V = \{u_i, \bar{u}_i, b_i, d_i : i \in [n]\} \cup \{y_j : j \in [m]\}$;

$E_a = \{\{u_i, b_i\}, \{b_i, \bar{u}_i\} : i \in [n]\} \cup \{\{u_i, d_i\}, \{d_i, \bar{u}_i\} : i \in [n]\} \cup$

$\{\{y_j, f(l_1^j)\}, \{y_j, f(l_2^j)\}, \{y_j, f(l_3^j)\} : C_j = (l_1^j \vee l_2^j \vee l_3^j), j \in [m]\}$;

$E_i = \{\{u, v\} : \forall u, v \in V \textit{ and } \{u, v\} \notin E_a\}$;

$E = E_a \cup E_i$;

where E_a is the set of addable edges in H, whereas each edge in E_i has infinite cost. Define $(d_v)_{v \in V}$ and $(g_v)_{v \in V}$ for each player:

- For all $i \in [n]$ and $\forall k \in \mathbb{N} \cup \{0\}$, set $g_{u_i}(k) = g_{\bar{u}_i}(k) = 30k$, $c_{u_i} = c_{\bar{u}_i} = 45$.
- For all $i \in [n]$, $\forall k \in \mathbb{N} \cup \{0\}$, set $g_{b_i}(k) = 30k + 2000n$ and $g_{d_i}(k) = 20k$, $c_{b_i} = c_{d_i} = 45$.
- For all $j \in [m]$ and $\forall k \in \mathbb{N} \cup \{0\}$, set $g_{y_j}(k) = 10k + 1000n$, $c_{y_j} = 30$.

Let the initial prosocial network H be empty, and the target profile $\mathbf{x}^* = (1, \cdots, 1)$ has all the players invest. The cost of adding an edge from set $\{\{y_j, f(l_1^j)\}, \{y_j, f(l_2^j)\}, \{y_j, f(l_3^j)\} : C_j = (l_1^j \vee l_2^j \vee l_3^j), j \in [m]\}$ and $\{\{u_i, d_i\}, \{d_i, \bar{u}_i\} : i \in [n]\}$ is 1. The cost of adding an edge from set $\{\{u_i, b_i\}, \{b_i, \bar{u}_i\} : i \in [n]\}$ is $c = 2n + 1$. The cost of adding edges in E_i is $+\infty$, meaning it can not be added in H. The modification budget B is set to be $(nc + n + m)$.

Next we prove that the instance of (3, B2)-SAT is a "YES" instance if and only if the instance of PNM with symmetric egalitarian is a "YES" instance, that is the target strategy profile $\mathbf{x}^*$ can be induced to be a PSNE by modification.

In one direction, let (3, B2)-SAT be a "YES" instance and its satisfying assignment be $F : \{l_i, \bar{l}_i : i \in [n]\} \to \{\text{TRUE}, \text{FALSE}\}$. We construct H in following steps:

(1) For each i, if l_i is TRUE, then we add edges $\{u_i, d_i\}, \{\bar{u}_i, b_i\}$ to H.
(2) For each i, if l_i is FALSE, then we add edges $\{u_i, b_i\}, \{\bar{u}_i, d_i\}$ to H.
(3) For each y_j, as (3, B2)-SAT be a "YES" instance, there must exist a literal $F(l_j) = \text{TRUE}$ in the clause C_j. We add edge $\{f(l_i), y_j\}$ to H.

The total cost of adding the above edges is $nc + n + m$. An example is shown on the right side of Fig. 2.

In the following, we prove that the target profile $\mathbf{x}^*$ is a PSNE.

- For each i, if l_i is TRUE, then $\{\bar{u}_i, b_i\} \in H$. So $\Delta U_{b_i}^{(F)}(k) = 60 > c_{b_i} = 45$, $\Delta U_{\bar{u}_i}^{(F)}(k) = 60 > c_{\bar{u}_i} = 45$. It means that b_i and $\bar{u}_i$ do not deviate from the strategy of 1. Also, as $\{u_i, d_i\} \in H$ and $g_{y_j}(k) > g_{d_i}(k)$, we have $\Delta U_{d_i}^{(F)}(k) = 50 > c_{d_i} = 45$, $\Delta U_{u_i}^{(F)}(k) = 50 > c_{u_i} = 45$, d_i and u_i also do not deviate. The analysis is similar when l_i is FALSE, and we can conclude that b_i, $\bar{u}_i$, d_i and u_i also do not deviate their strategies.
- For each j, because the (3, B2)-SAT instance is a "YES" instance, there must exist one TRUE literal l_j in clause C_j. Therefore, each y_j must have a neighbor literal node $f(l_j)$, denoted by u_j w.l.o.g. We have $\Delta U_{y_j}^{(F)}(k) = 40 > c_{y_j} = 30$, y_j does not deviate from playing 1.

Hence the instance of PNM with symmetric egalitarianism is a "YES" instance.

In the other direction, let PNM with symmetric egalitarianism be a "YES" instance. Then we can characterize the ultimate prosocial network H as:

(1) First, because $\Delta g_{b_i}(k) < c_{b_i}$ for each b_i, thus at least one of $\{u_i, b_i\}$ and $\{\bar{u}_i, b_i\}$ in H. Meanwhile, note that $C(\{b_i, u_i\}) = 2n+1$ and the total budget $B = n(2n+1)+n+m < (n+1)(2n+1)$. Thus for each b_i, only one of $\{u_i, b_i\}$ and $\{\bar{u}_i, b_i\}$ is in H.
(2) Second, because the rest of the budget is $n + m$, and there are exactly n number of d_i and m number of y_j which should be adjacent to the edges in H. Otherwise, they will deviate from playing 1. Hence, each d_i and y_j must exactly be adjacent to one edge in H.
(3) Finally, if $\{u_i, y_j\}$ is added in H, then $\{u_i, d_i\}$ must be added in H too. Otherwise u_i will deviate from playing 1. Meanwhile, u_i will also deviate from playing 1 when she has no neighbors in H. Hence, for each $\{u_i, b_i\} \notin H$, we have $\{u_i, d_i\} \in H$.

Now consider the following assignment $F : \{l_i, \bar{l}_i : i \in [n]\} \to \{\text{TRUE}, \text{FALSE}\}$ of (3,B2)-SAT instance. For all $\{u_i, d_i\} \in H$ we have $F(l_i) = \text{TRUE}$ and for all $\{u_i, b_i\} \in H$, we have $F(l_i) = \text{FALSE}$. We claim that F is a satisfying assignment. If there is a clause $C_j = (l_1^j \vee l_2^j \vee l_3^j)$ which is not satisfied, meaning that all $\{f(l_1^j), y_j\}, \{f(l_1^j), y_j\}, \{f(l_1^j), y_j\}$ are not in H. It forces y_j to deviate from playing 1, contradicting the assumption that $\mathbf{x}^*$ is a PSNE. Hence F is a satisfying assignment, and the instance of (3,B2)-SAT is a "YES" instance. □

Note that in the above proof, the target profile where all players invest is the only optimizer of social welfare $SW(\mathbf{x})$. This implies that inducing strategies to optimize social welfare is also, in general, NP-complete.

Although the general PNM problem with symmetric egalitarianism is NP-complete, we next show that PNM is tractable when the underlying relation graph G has the special structure of a tree.

Theorem 7. *For any fixed target profile, if the relation graph G is a tree, then the PNM problem with symmetric egalitarianism is polynomial-time solvable.*

Proof sketch. We also design a dynamic algorithm in a bottom-up manner to solve this PNM problem. In this algorithm, each leaf or internal node passes a *minimum cost table* to its parent. Consider a node v with its parent u. A pair related to edge $\{v, u\}$ in minimum cost table T_v is denoted by $(b_{\{v,u\}}, c_{\{v,u\}})$, where $b_{\{v,u\}} \in \{0, 1\}$, representing edge $\{v, u\}$ is in H' or not, and $c_{\{v,u\}}$ representing the minimum cost of the subtree rooted at v, which makes the nodes in this subtree not deviate from strategy profile $\mathbf{x}^*$. The minimum cost table only contains the pairs whose $c_{\{v,u\}} \leq B$. Once a vertex v is processed, if the cost $c_{\{v,u\}} > B$, then we conclude that the PSNE does not exist in this game. Otherwise, we can get a modification scheme to change H into H' and such that the target strategy profile becomes a PSNE. The full proof is in Appendix C. □

For the case where egalitarianism is asymmetric, it is proven to be polynomial-time solvable, because a modification on edge is one-way, only affecting one player's extent of egalitarianism. We present the following theorem and its proof is in Appendix D.

Theorem 8. *For any fixed target profile, the PNM problem with asymmetric egalitarianism is polynomial-time solvable.*

5.2 Results for PNM problem with altruism

For the PNM problem with altruism, previous works mainly focus on the target strategy profile where all players invest. [47] and [35] respectively proved that the PNM problem is NP-complete even when the relation graph is a clique or a tree. However, for the target strategy profile maximizing social welfare, the hardness results for the PNM problem with altruism have not been explored yet. We first prove that this PNM problem is intractable when the relation graph is a tree by reducing the PNM problem from the KNAPSACK problem. The full proof of this result is in Appendix E.

Theorem 9. *For the target profile where the social welfare is maximized, PNM with symmetric or asymmetric altruism is NP-hard when the input network is a tree.*

For the case where the relation graph is a clique, we also show it is intractable by similarly reducing the problem from the KNAPSACK problem. We present the following theorem, and its proof is in Appendix F.

Theorem 10. *For the target profile where the social welfare is maximized, PNM with symmetric or asymmetric altruism is NP-hard when the input network is a clique.*

Based on the above theorems, we know that when the target profile is the optimal solution for social welfare, PNM problem with altruism is intractable in many scenarios. However, the following result shows that for any target profile $\mathbf{x}^*$ in which all players invest or maximize the social welfare, once it can be induced to a PSNE in polynomial time for PNM problem with egalitarianism, then it is also solvable in polynomial time for PNM problem with altruism.

Theorem 11. *Given a target strategy profile $\boldsymbol{x}^*$ in which all players invest or maximize the social welfare, if we can induce $\boldsymbol{x}^*$ to be a PSNE for PNM problem with egalitarianism in a polynomial time, then $\boldsymbol{x}^*$ also can become to be a PSNE for PNM problem with altruism in a polynomial time.*

proof sketch. To obtain this claim, we shall prove that the modification solution for the PNM problem with egalitarianism is just the solution for the PNM problem with altruism. The full proof is in Appendix G. □

5.3 Results for PNM problem with collectivism

For the PNM problem with collectivism, we begin by showing that the problem is tractable for the target profile where all players invest.

Theorem 12. *For the target strategy profile $\mathbf{x}^* = (1, \cdots, 1)$, the PNM problem with symmetric and asymmetric collectivism is polynomial-time solvable.*

proof skecth. As $H = G$, we can not add edges into H for the PNM problem, while deleting edges cannot increase player willingness to invest. Therefore, solving the PNM problem with symmetric collectivism is equivalent to checking whether all players invest is a PSNE in the original BNPG game. The full proof is in Appendix H. □

By Theorem 4, that the strategy profile maximizing social welfare is also a PSNE, we can conclude that the PNM problem with collectivism is also tractable.

Theorem 13. *For the target strategy profile maximizing social welfare, the PNM problem with symmetric and asymmetric collectivism is polynomial-time solvable.*

6 CONCLUSION AND FUTURE WORK

In this paper, we study the BNPG games from the perspective of prosociality. Three types of prosociality, altruism, collectivism, and egalitarianism, are discussed in detail. For these models, we first develop the computational complexity issues of deciding the existence of PSNE. We prove the hardness of the problem for general networks, but for some special networks like tree and clique, we show tractability results and provide polynomial time algorithms. We also spend a lot of effort studying the PNM problem with prosociality. For this problem, we are more interested in inducing the target strategy profile where all players invest or maximize the social welfare to be a PSNE by modifying the prosocial graph. For all three types, we completely characterize their corresponding computational complexity results. These results enable policymakers to strategically run campaigns to encourage people's participation.

Our work also leaves some questions open. One important research direction is to study the computational complexity of finding mixed strategy equilibrium. Since [35] proved PPAD-hardness in BNPG games with altruism, we are more interested in the computational complexity of other prosocial types in BNPG games. Another interesting future work is to explore the parameterized complexity of BNPG games with different prosocial types.

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APPENDIX

A PROOF OF THEOREM 2

Proof. We give a constructive proof for this theorem. Given a tree G, we denote its root by R (the choice of the root is arbitrary). Nodes in G are categorized into three classes: leaf nodes, internal nodes, and the root. Our algorithm run in the backward pass, meaning that we traverse the nodes in a depth-first order (start with the leaves and end at the root). Each leaf or internal node passes a table to its parent. We call the table *conditional satisfiable table* since it contains all satisfiable situation conditioned on its parent's strategy. Our algorithm have the following three parts:

Leaf Nodes: Suppose v is a leaf node and u is its parent. The algorithm compute all tuple (x_u, n_u, x_v, n_v) satisfied:

(1) $n_v = x_u$: since v does not have any child, its only neighbor is parent u.
(2) When $\{u, v\} \in H$: if $x_v = 0$, the tuple should satisfied $\Delta g_v(n_v) + a\Delta g_u(x_u + n_u) < c_v$; else if $x_v = 1$, the condition is $\Delta g_v(n_v) + a\Delta g_u(x_u + n_u - 1) > c_v$: therefore v will not deviate from x_v.
(3) When $\{u, v\} \notin H$, if $x_v = 0$, the tuple should satisfied $\Delta g_v(n_v) < c_v$; else if $x_v = 1$, the condition is $\Delta g_v(n_v) > c_v$: such that v will not deviate too.
(4) $x_v \le n_u \le x_v + d_u - 1$: the limiting condition of possible neighbor number for node u.

The algorithm then put the satisfied tuples into the table T_v. Computing the table of leaf nodes take $O(2d_{max} * |V|)$ time since there are at most $|V| - 1$ numbers of leaf nodes and each node have at most $2d_{max}$ satisfiable tuple. If $T_v = \emptyset$ for some leaf v, we conclude a PSNE does not exist for this game.

Internal Nodes: Now we consider a internal node v. Suppose its parent is u and it has p children $w_1, ..., w_p$. For the purpose of induction, suppose the conditional best-response tables $T_{w_1}, ..., T_{w_p}$ from $w_1, ..., w_p$ have been passed to v. We define $N_v = \{w_i : \{w_i, v\} \in H\} \cup \{u : \{u, v\} \in H\}$ be the set of all v's neighbor in the prosocial grape. Now we traverse each tuple (x_v, n_v, x_u, n_u): $x_v, x_u \in \{0, 1\}$, $n_v, n_u \in \{0, 1, ..., d_{max}\}$. To simplify our proof, we define a temporary table $T_u = \{(x_u, n_u, x_v, n_v)\}$. Then we divide the children into three categories:

$$\alpha_v(n_v) := \{w_i : \exists(1, n_{w_i}, x_v, n_v) \in T_{w_i} \textit{ and } \forall(0, n_{w_i}, x_v, n_v) \notin T_{w_i}\}$$

$$\beta_v(n_v) := \{w_i : \forall(1, n_{w_i}, x_v, n_v) \notin T_{w_i} \textit{ and } \exists(0, n_{w_i}, x_v, n_v) \in T_{w_i}\}$$

$$\gamma_v(n_v) := \{w_i : \exists(1, n_{w_i}, x_v, n_v) \in T_{w_i} \textit{ and } \exists(0, n_{w_i}, x_v, n_v) \in T_{w_i}\}$$

The conditional satisfiable table should contain tuples satisfy following conditions:

(1) $|\alpha(v)| + x_u \le n_v \le |\alpha(v)| + |\gamma(v)| + x_u$: the limiting condition of possible neighbor for node v.
(2) Exist one possible equilibrium in the subtree of v, such that v will not deviate from x_v. When $x_v = 1$, it can be check with Algorithm 5, which is a traversal algorithm run in $O(d_{max}^3 \log(d_{max}))$ times.
(3) When $x_v = 0$, the algorithm is almost the same with algorithm 5, while for all $i = \{v, u, w_1, ..., w_p\}$, the function $g_i(k)$ should be replaced by $g_i(k+1)$, and the final condition $a(Max\_min+Add) \ge c_v$ should be replaced by $a(Max\_min+Add) \le c_v$.

There are three important component in Algorithm 5:

- *Tuple Sorting*: In this phase, on condition with the same x_{w_i}, we sort n_{w_i} for each w_i. The full process is in Algorithm 2.

Algorithm 2: Tuple Sorting
```
Input: {T_{w_1}, ..., T_{w_p}}
Output: {A_i, B_i, C_i}_{i∈[p]}
for i ∈ [p] and w_i ∈ N_v do
    if w_i ∉ γ_v(n_v) then
        sort all (x_{w_i}, n_{w_i}, x_v, n_v) ∈ T_{w_i} by n_{w_i} in decending
          order and get array A_i
    end
    if w_i ∈ γ_v(n_v) then
        sort all (0, n_{w_i}, x_v, n_v) ∈ T_{w_i} by n_{w_i} in decending
          order and get array B_i ;
        sort all (1, n_{w_i}, x_v, n_v) ∈ T_{w_i} by n_{w_i} in decending
          order and get array C_i
    end
end
return {A_i, B_i, C_i}_{i∈[p]}
```

- *Min Children*: In this phase, we consider the case when x_v deviate from 1, calculating the minimum utility of v's children in this situation. The full process is in Algorithm 3.

Algorithm 3: Min Children
```
Input: {A_i, B_i, C_i}_{i∈[p]}, N_v, {g_{w_1}, ..., g_{w_p}}
Output: {a_i, b_i, c_i}_{i∈[p]}
for i ∈ [p] and w_i ∈ N_v do
    if w_i ∉ γ_v(n_v) then
        get (x_{w_i}, n_{w_i}, x_v, n_v) which is the last element in A_i
          and calculate a_i = g_{w_i}(x_{w_i} + n_{w_i} − 1)
    end
    if w_i ∈ γ_v(n_v) then
        get (0, n_{w_i}, x_v, n_v) which is the last element in B_i
          and calculate b_i = g_{w_i}(n_{w_i} − 1) ;
        get (1, n_{w_i}, x_v, n_v) which is the last element in C_i
          and calculate c_i = g_{w_i}(n_{w_i})
    end
end
return {a_i, b_i, c_i}_{i∈[p]}
```

- *Max Add*: In this phase, we come back to $x_v = 1$, and checking if there exist and equilibrium in the subtree of v, such that x_v also will not deviate from 1. The full process is in Algorithm 4.

Root: As the properties introduced above, there exists a PSNE for the whole tree if and only if the conditional satisfiable table for root r is not empty. Therefore, we add a imaginary parent node r^* for r and calculate the conditional satisfiable table for r in Algorithm 5.

Finally, if the table of root r is not empty, we claim that there exist a PSNE in this game. Otherwise, we conclude a PSNE does not exist for this game. □

Algorithm 5: Tuple Stable Determination

```
Input: (x_v, n_v, x_u, n_u), N_v, α_v(n_v), β_v(n_v), γ_v(n_v),
       {g_v, g_u, g_{w_1}, ..., g_{w_p}}, {T_{w_1}, ..., T_{w_p}}
Output: YES / No
{A_i, B_i, C_i}_{i∈[p]} ← Tuple Sorting(T_{w_1}, ..., T_{w_p}) ;
while ∀A_i, B_i, C_i is not empty do
    {a_i, b_i, c_i}_{i∈[p]} ← Min children({A_i, B_i, C_i}_{i∈[p]});
    Add ← min({a_i, b_i, c_i}_{i∈[p]}) ;
    w_j = arg min({a_i, b_i, c_i}_{i∈[p]}) ;
    Max_min ← +∞ ;
    N_1 ← n_v − (|γ(v)| − |γ(v) ∩ N_v|) − |α(v)| − x_u ;
    N_2 ← n_v − |α(v)| − x_u ;
    if w_j ∈ γ_v(n_v) and x_{w_j} = 1 then
        N_1 ← N_1 − 1 and N_2 ← N_2 − 1;
        if N_2 < 0 then
            delete (x_{w_i}, n_{w_i}, x_v, n_v) in {A_i, B_i, C_i} ;
            continue ;
        end
    end
    for i ∈ [p]/{j} and w_i ∈ N_v/γ_v(n_v) do
        (x_{w_i}, n_{w_i}, x_v, n_v) = pop(A_i) ;
        Max_min ← min(Max_min, g_{w_i}(x_{w_i} + n_{w_i}))
    end
    Max ← {} ;
    for i ∈ [p]/{j} and w_i ∈ N_v ∩ γ_v(n_v) do
        (0, n_{w_i}, x_v, n_v) = pop(B_i) ;
        (1, n'_{w_i}, x_v, n_v) = pop(C_i) ;
        Append (0, 0, w_i, g_{w_i}(n_{w_i})) to Max ;
        Append (0, 1, w_i, g_{w_i}(n'_{w_i} + 1)) to Max ;
    end
    Sort Max by the third component in abscending order ;
    Max_add ← Max Add(N_1, N_2, Max_add, Max) ;
    Max_min ← min(Max_min, g_u(n_u + x_u), Max_add) ;
    Add ← min(Add, g_u(n_u + x_u − 1)) ;
    if a(Max_min + Add) ≥ c_v then
        return TRUE
    end
end
return FALSE
```

Algorithm 4: Max Add

```
Input: N_1, N_2, Max_add, {Max}
Output: Max_add
for N_1 ≤ n ≤ N_2 do
    num ← 0 ;
    for (s_{w_i}, x_{w_i}, w_i, g_{w_i}) ∈ Max do
        if s_{w_i} = 1 or num == n then
            Max_add ← max(Max_add, g_{w_i}) ;
            break ;
        end
        if x_{w_i} = 1 then
            num += 1 ;
        end
        Find (s'_{w_i}, 1 − x_{w_i}, w_i, g'_{w_i}) ∈ Max and set s'_{w_i} = 1.
    end
end
return Max_add
```

B PROOF OF THEOREM 3

Proof. Let us consider an equilibrium profile in which there are exactly p players investing the public goods. Under this equilibrium, we have $x_{v_i}+n_{v_i} = p$ for each $i \in [n]$, as the relation graph is a clique. Then, for each player v_i, define $\Delta U_{v_i}^{(F)}(k) = g_{v_i}(k+1) - g_{v_i}(k)$. Now we can divide the players into the following four categories:

$$\alpha(p) := \{v_i \in V : \Delta U_{v_i}^{(F)}(p-1) \geq c_{v_i} \text{ and } \Delta U_{v_i}^{(F)}(p) > c_{v_i}\}$$

$$\beta(p) := \{v_i \in V : \Delta U_{v_i}^{(F)}(p-1) < c_{v_i} \text{ and } \Delta U_{v_i}^{(F)}(p) \leq c_{v_i}\}$$

$$\gamma(p) := \{v_i \in V : \Delta U_{v_i}^{(F)}(p-1) \geq c_{v_i} \text{ and } \Delta U_{v_i}^{(F)}(p) \leq c_{v_i}\}$$

$$\delta(p) := \{v_i \in V : \Delta U_{v_i}^{(F)}(p-1) < c_{v_i} \text{ and } \Delta U_{v_i}^{(F)}(p) > c_{v_i}\}$$

As the marginal utility of v_i is larger than the investment cost, each player in $\alpha(p)$ does not deviate from the strategy of "investing" when there are $p-1$ or p other players invest. We can get the similar observations for the sets $\beta(p), \gamma(p)$ and $\delta(p)$. Based on these observations, we claim that there exists a PSNE if and only if $|\alpha(p)| \leq p$, $|\beta(p)| \leq n-p$, and $|\delta(p)| = 0$.

In one direction, suppose there is an equilibrium profile $\mathbf{x}^*$ in which exactly p players invest. First we conclude that no player in $\alpha(p)$ would like to play 0. Otherwise, the player who does not invest can obtain more utility by deviating from strategy 0, since $\Delta U_{v_i}^{(F)}(p) > c_{v_i}$ for all $v_i \in \alpha(p)$. This contradicts the assumption that $\mathbf{x}^*$ is an equilibrium profile. Thus all players in $\alpha(p)$ invest and $|\alpha(p)| \leq p$. Similarly, we can conclude that all players in $\beta(p)$ play 0. Otherwise, any player who invests in $\beta(p)$ can benefit more by deviating from strategy 1. Therefore $\mathbf{x}^*$ is not a PSNE. As there are exactly p players playing 1 in $\mathbf{x}^*$, implying the number of players not investing is $n-p$, we have $|\beta(p)| \leq n-p$. Finally, if there is a player in $\delta(p)$, then whatever she invests or not, deviating her current strategy can bring her more utility, and thus $\mathbf{x}^*$ is not an equilibrium. So $|\delta(p)| = 0$.

In the other direction, consider the case in which $|\alpha(p)| \leq p$, $|\beta(p)| \leq n-p$, and $|\delta(p)| = 0$. Let us construct an equilibrium profile $\mathbf{x}^* = (x_{v_i})_{v_i \in V}$ as follows. First, set $x_{v_i} = 1$, for all $v_i \in \alpha(p)$, and set $x_{v_i} = 0$, for all $v_i \in \beta(p)$. The rest of players are all in $\gamma(p)$, because $|\delta(p)| = 0$. As $|\alpha(p)| \leq p$ and $|\beta(p)| \leq n-p$, we arbitrary select some players to constitute a subset $\mathcal{I} \subseteq \gamma(p)$, such that $|\mathcal{I}| = p - |\alpha(p)| \geq 0$. Then for each $v \in \mathcal{I}$, we set $x_v = 1$ and for others $v \in \gamma(k)\backslash\mathcal{I}$, we set $x_v = 0$. By the construction of $\mathbf{x}^*$, we claim that $\mathbf{x}^*$ is a PSNE. For each v_i who plays 0, she may be in $\beta(p)$ or in $\gamma(p)$, then $\Delta U_{v_i}^{(F)}(p) \leq c_{v_i}$, implying that v_i won't deviate. Similarly, for any v_i who invests, she may be in $\alpha(p)$ or in $\gamma(p)$, thus $\Delta U_{v_i}^{(F)}(p-1) \geq c_{v_i}$, and v_i also won't deviate. Hence, under the strategy profile $\mathbf{x}^*$, nobody would like to deviate her current strategym and thus $\mathbf{x}^*$ is a PSNE.

In all, we now present a polynomial time algorithm to check the existence of PSNE. First, for $p = 0$, let us compute $\Delta U_{v_i}^{(F)}$. If $\Delta U_{v_i}^{(F)} \leq c_{v_i}$ for all $v_i \in V$, then $\mathbf{x}^* = (0, 0, \cdots, 0)$ is an equilibrium.

Algorithm 6: Compute minimum cost table for leaf v

```
Input: Δg_v, Δg_u, c_v, a, C({v,u}), B, H, x*
Output: T_v
T_v ← {} ;
if x_v = 0 then
    if {v,u} ∈ H then
        if Δg_v(x_v + x_u) + aΔg_u(x_u + n_u) ≤ c_v then
            put (1,0) into T_v ;        // Retain {v,u} ∈ H′
        end
        else if Δg_v(x_v + x_u) ≤ c_v and C({v,u}) ≤ B then
            put (0, C({v,u})) into T_v ;      // Delete {v,u}
        end
    end
    if {v,u} ∉ H then
        if Δg_v(x_v + x_u) ≤ c_v then
            put (0,0) into T_v ;      // Maintain {v,u} ∉ H′
        end
    end
end
if x_v = 1 then
    if {v,u} ∈ H then
        if Δg_v(x_u) + aΔg_u(x_u + n_u − 1) ≥ c_v then
            put (1,0) into T_v ;        // Retain {v,u} ∈ H′
        end
    end
    if {v,u} ∉ H then
        if Δg_v(x_u) ≥ c_v then
            put (0,0) into T_v ;      // Maintain {v,u} ∉ H′
        end
        else if Δg_v(x_u) + aΔg_u(x_u + n_u − 1) ≥ c_v and
         C({v,u}) ≤ B then
            put (1, C({v,u})) into T_v ;          // Add {v,u}
        end
    end
end
if T_v = ∅ then
    return No PSNE exists ;
end
else
    return T_v ;
end
```

Algorithm 7: Inevitable Cost

```
Input: T_{w_1}, ..., T_{w_p}
Output: C_init
C_init ← 0 ;
for w_i ∈ α(v) ∪ β(v) do
    for {c_x : (., c_x) ∈ T_{w_i}} do
        C_init = C_init + c_x
    end
end
return C_init
```

Algorithm 8: Compute minimum cost table for a internal node

```
Input: g_v, g_u, g_{w_1}, ..., g_{w_p}, c_v, a, C(e), B, H, T_{w_1}, ..., T_{w_p}, x*
Output: T_v
M_0 ← min_{w_i ∈ α(v)} g_{w_i}(x_{w_i} + n_{w_i}) ;
M_1 ← min_{w_i ∈ α(v)} g_{w_i}(x_{w_i} + n_{w_i} + 1) ;
C_init ← Inevitable Cost(T_{w_1}, ..., T_{w_p}) ;
C^0_min ← B + 1 ;
for w_i ∈ γ(v) do
    M^0_b = min (g_{w_i}(x_{w_i} + n_{w_i}), M_0) ;
    M^1_b = min (g_{w_i}(x_{w_i} + n_{w_i} + 1), M_1) ;
    if Δg_v(x_v + n_v) + a(M^1_b − M^0_b) ≤ c_v then
        C_sum ← 0 ;
        for w_j ∈ γ(v) and i ≠ j do
            if M^0_b ≤ g_{w_j}(x_{w_j} + n_{w_j}) then
                C_sum+ = min(c^0_{w_j}, c^1_{w_j})
            end
            else
                C_sum+ = c^0_{w_j}
            end
        end
    end
    else
        C_sum ← B + 1 ;
        l ← i ;
        for w_j ∈ γ(v) and j ≠ i do
            if M^0_b ≤ g_{w_j}(x_{w_j} + n_{w_j}) and
             Δg_v(x_v + n_v) + a(g_{w_j}(x_{w_j} + n_{w_j} + 1) − M^0_b) ≤ c_v
             then
                C_sum ← min(C_sum, c^1_{w_j}) ;
                l ← j ;
            end
        end
        for w_j ∈ γ(v) and j ≠ i, j ≠ l do
            if M^0_b ≤ g_{w_j}(x_{w_j} + n_{w_j}) then
                C_sum+ = min(c^0_{w_j}, c^1_{w_j})
            end
            else
                C_sum+ = c^0_{w_j}
            end
        end
    end
    C_min = min(C_min, C_init + C_sum)
end
if C_min ≤ B then
    put (0, C_min) to T_v ;
end
```

For each $p = 1, \cdots, n$, we compute $\Delta U_{v_i}^{(F)}(p)$, and construct $\alpha(p)$, $\beta(p)$, $\gamma(p)$ and $\delta(p)$, respectively. If there exists one p, such that $|\alpha(p)| \leq p$, $|\beta(p)| \leq n - p$ and $|\delta(p)| = 0$, then a PSNE is obtained. Otherwise, we conclude that no PSNE exists in this BNPG game with egalitarianism. As for the time complexity of this algorithm, it takes $O(|V|)$ time to compute all $\Delta U_{v_i}^{(F)}(p)$ for a given p. So the problem of checking the existence of PSNE in the BNPG game with egalitarianism is $O(|V|^2)$ time solvable, if G is a clique. □

C PROOF OF THEOREM 7

Theorem 14. *For any fixed target profile, PNM problem with symmetric egalitarian is polynomial-time solvable when the input network is a tree.*

Proof. We show a recursive algorithm for this theorem. For a target profile $\mathbf{x}^*$, each player v_i's strategy is denoted by x_{v_i} and each v_i's investing neighbor number is denoted by n_{v_i}. Our goal is to modify the prosocial graph H to a graph H' within budget B, and make $\mathbf{x}^*$ be an equilibrium. For the tree G, we denote its root by node R (the root is chosen arbitrary). The nodes in G are categorized into three classes: leaf nodes, internal nodes, and the root. Our algorithm traverse the nodes in a depth-first order, leafs are the first and root is the final. Each leaf or internal node v_i passes a table T_{v_i} to its parent. We call the table *minimum cost table* since it record the minimum cost of adding or removing edges into the subtree. A pair in minimum cost table is denoted by $(x_{\{v,u\}}, c_{\{v,u\}})$, where $x_{\{v,u\}} \in \{0, 1\}$, representing the edge $\{v, u\}$ added in H' or not, and $c_{\{v,u\}}$ representing the minimum cost in the subtree which make the node in subtree not deviate from $\mathbf{x}^*$. To simplify our proof, we also define $\Delta g_{v_i}(k) = g_{v_i}(k+1) - g_{v_i}(k)$.

Leaf node: Suppose v is a leaf node and u is its parent. Since v does not have any child, there are only two possible scenarios in the equilibrium: $\{v, u\} \in H'$ or $\{v, u\} \notin H'$. Thus, computing the table is equivalent to calculate the minimum cost such that node v will not deviate from x_v. The procedure to compute the minimum cost table for a leaf node v is summarized in Algorithm 6.

Internal node: For each internal node v with parent u, suppose it has p children $w_1, ..., w_p$. For the purpose of induction, suppose the minimum cost tables $T_{w_1}, ..., T_{w_p}$ have been passed to v. Now we can divide the children into the following three categories:

$$\alpha(v) := \{w_i : (1, .) \in T_{w_i} \text{ and } (0, .) \notin T_{w_i}\}$$
$$\beta(v) := \{w_i : (0, .) \in T_{w_i} \text{ and } (1, .) \notin T_{w_i}\}$$
$$\gamma(v) := \{w_i : (1, .) \in T_{w_i} \text{ and } (0, .) \in T_{w_i}\}$$

For the convenience of algorithm description, we define $c_{w_i}^0, c_{w_i}^1$ to denote the minimum cost of w_i, i.e., $(0, c_{w_i}^0), (1, c_{w_i}^1) \in T_{w_i}$ for all $w_i \in \gamma(v)$. The algorithm that compute the minimum cost table for the internal node v when $x_v = 0$ is showed in Algorithm 8. When $x_v = 0$, the algorithm is almost the same with algorithm 8, while for all $i = \{v, u, w_1, ..., w_p\}$, the function $g_i(k)$ should be replaced by $g_i(k-1)$. Meanwhile all "If" condition (except $C_{min} \leq B$) should negate the judgment symbol.

Root: As the properties introduced above, there exists a PSNE for the game if and only if there exists $(0, c_r^0)$ in the minimum cost table of r. Therefore, we add a imaginary parent node r^* for root r, and also run algorithm 8. Finally, if the table of root r is not empty, we claim that the PNM problem is solvable. Otherwise, we conclude that PNM is not solvable. □

D PROOF OF THEOREM 8

Proof. Let $(G, H, (g_i)_{i \in V}, (c_i)_{i \in V}, a, C, B, \mathbf{x}^*)$ be a instance of PNM problem with asymmetric egalitarian. Let N_v denote the set of v's neighbors in G. Because H is a directed graph, when we add/remove an edge (u, v) in H, only the utility of v will change, while the utility of any other nodes will not change. Thus, we can discuss the edge's modification for each node, calculate the minimum cost, and finally determine if total costs exceed budget.

Now we consider for each player $v_i \in V$. When the target strategy $x_i = 0$, v_i will not deviate from playing 0 only if

$$g_v(1 + n_v) + a \min_{(u,v) \in H} g_u(x_u + n_u + 1) - c_v$$
$$\leq g_v(n_v) + a \min_{(u',v) \in H} g_{u'}(x_{u'} + n_{u'}).$$

Because n_v, x_u, n_u, c_v are constants, this condition can be written as

$$\min_{(u,v) \in H} g_u(x_u + n_u + 1) - \min_{(u',v) \in H} g_{u'}(x_{u'} + n_{u'}) \tag{6}$$
$$\leq \frac{1}{a}(g_v(n_v) - g_v(1 + n_v) + c_v) \tag{7}$$

Thus, we should only find the minimum cost modifying sets, such that the neighbors' utility gap between $x_v = 0$ and $x_v = 1$ is less than (7). We propose a traverse algorithm 9 to solve this problem.

When $x_v = 1$, the algorithm is almost the same with algorithm 9, while for all $i = v \in N_v$, the function $g_i(k)$ should be replaced by $g_i(k-1)$. and the "If" determinate condition $g_{v_k}(x_{v_k} + n_{v_k} + 1) \leq M_b + \frac{1}{a}(c_{v_i} - g_{v_i}(1 + n_{v_i}) + g_{v_i}(n_{v_i}))$ should be reverse to $g_{v_k}(x_{v_k} + n_{v_k} + 1) \geq M_b + \frac{1}{a}(c_{v_i} - g_{v_i}(1 + n_{v_i}) + g_{v_i}(n_{v_i}))$.

Finally, if $\sum_{v \in V} C_v \leq B$, we conclude that this PNM problem have a solution modification $\Cup_{v \in V} E_v$. Otherwise, we conclude that the PNM problem is not solvable.

□

E PROOF OF THEOREM 9

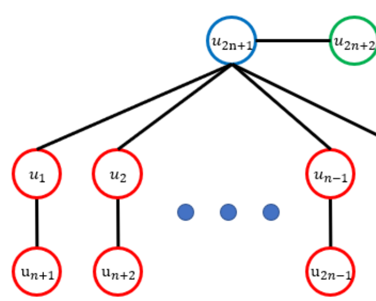

Figure 3: Example Graph for KNAPSACK in Theorem 9

Proof. We reduce the PNM problem with asymmetric altruism from the KNAPSACK problem. Given a knapsack of capacity W, a targeted value V, a set of items $1, \cdots, n$ with values $v_1, \cdots, v_n$ and weights $w_1, \cdots, w_n$, our aim is to check whether there exists a subset S of items such that $\sum_{i \in S} w_i \leq W$ and $\sum_{i \in S} v_i \geq V$.

Algorithm 9: Compute edge modification for v

Input: $g_{v_1}, ..., g_{v_n}, c_v, a, C(e), B, H, ,\mathbf{x}^*, V$
Output: C_v, E_v
$C_v \leftarrow B + 1$;
$E_v \leftarrow \{\}$;
for $v_j \in N_{v_i}$ **do**
  $E_j \leftarrow \{\}$;
  $e_j \leftarrow null$;
  $M_b = g_{v_j}(x_{v_j} + n_{v_j})$;
  $C_{min} \leftarrow B + 1$;
  $C_{sum} \leftarrow 0$;
  if $(v_j, v_i) \in H$ **then**
    $C_{sum} = C((v_j, v_i))$;
  end
  for $v_k \in N_{v_i}/\{j\}$ **do**
    if $g_{v_k}(x_{v_k} + n_{v_k}) < M_b$ **then**
      if $(v_k, v_i) \in H$ **then**
        $C_{sum} = C_{sum} + C((v_k, v_i))$;
        append (v_k, v_i) to E_j ;
      end
    end
    else if $g_{v_k}(x_{v_k} + n_{v_k} + 1) \leq M_b + \frac{1}{a}(c_{v_i} - g_{v_i}(1 + n_{v_i}) + g_{v_i}(n_{v_i}))$ **then**
      if $(v_k, v_i) \in H$ **then**
        $C_{min} \leftarrow 0$;
        $e_j \leftarrow null$;
      end
      else
        if $C((v_k, v_i)) < C_{min}$ **then**
          $C_{min} \leftarrow C((v_k, v_i))$;
          $e_j \leftarrow (v_k, v_i)$;
        end
      end
    end
  end
  if $C_{sum} + C_{min} < C_v$ **then**
    $C_v \leftarrow C_{sum} + C_{min}$;
    $E_v \leftarrow E_j \cup \{e_j\}$;
  end
end
return C_v, E_v

Construct an instance of PNM with asymmetric altruism. The relation network $G = (V, E)$ is defined as follows:

$V = \{u_i : i \in [2n + 2]\}$

$E = \{\{u_i, u_{n+i}\} : i \in [n]\} \cup \{\{u_i, u_{2n+1}\} : i \in [n]\} \cup \{u_{2n+1}, u_{2n+2}\}$

Let $a = 1$ and the prosocial graph $H\,(V, E')$ be empty. Set the target profile $\mathbf{x}^*$ to be the one where all players invest. This target profile turns out to be the only maximizer of social welfare in our construction. We will show the equivalence in later proof. Now we define the functions $g_u(.)$. Set $g_{u_{2n+1}}(x) = 0$ and $g_{u_{2n+2}}(x) = x \cdot 2V$ for all $x \geqslant 0$. For all $i \in [n]$, $g_{u_i}(x) = x \cdot v_i$ for all $x \geqslant 0$. For all $i \in [n]$, $g_{u_{n+i}}(x) = x \cdot V$ for all $x \geqslant 0$. For all $i \in [2n+2]$, $c_{u_i} = V$. The cost of introducing the edge (u_{2n+1}, u_i) is w_i for $i \in [n]$. The cost of introducing the edge (u_{2n+1}, u_{2n+2}) and (u_{2n+2}, u_{2n+1}) is $2W$. For remaining edges e, cost of adding e is 0. Let the total budget be W. The reduction clearly can be constructed in polynomial time.

Now we prove that in the construction above, the profile where all players invest maximizes social welfare. To maximize the social welfare, for $i \in [n]$, as u_i is connected to u_{n+i} and u_{2n+1}, so if u_i invests, the social welfare is changed by $V + v_i - V > 0$, so u_i must invest; Similarly, for $i \in [n]$, as u_{n+i} is connected to u_i, so if u_{n+i} invests, the social welfare is changed by $V + v_i - V > 0$, so u_{n+i} must invest; For u_{2n+2}, as u_{2n+2} is connected to u_{2n+1}, so if u_{2n+2} invests, the social welfare is changed by $2V - V > 0$, so u_{2n+2} must invest. For u_{2n+1}, as u_{2n+1} is connected to u_{2n+2} all u_i for $i \in [n]$, so if u_{2n+1} invests, the social welfare is changed by $\sum_{i=1}^{n} v_i + 2V - V > 0$, so u_{n+i} must invest. Then the target profile where the social welfare is maximized is equal to the target profile where all players invest.

Now we show that KNAPSACK Problem is a “YES” instance if and only if PNM with asymmetric altruism is a “YES” instance.

First, suppose that there is a set S of items that solves the KNAPSACK problem. Then, we introduce all the edges (u_{2n+1}, u_i) for $i \in S$. Each node u_i will invest and the cost of introducing these edges will not exceed the budget W.

For the converse direction, let the PNM with asymmetric altruism be a “YES” instance. Set the set of edges introduced as S'. Let $S := \{i : (u_{2n+1}, u_i) \in S'\}$. Hence if the subset S of items is chosen then we have $\sum_{i \in S} v_i \geqslant V$ and $\sum_{i \in S} w_i \leqslant W$. Hence the KNAPSACK problem is “YES” instance.

The prove of PNM with symmetric altruism is almost the same. We reduce it to the same KNAPSACK problem, with the only difference being the definition of edge adding / removing. We now redefine it for this situation. The cost of introducing the edge $\{u_{2n+1}, u_i\}$ is w_i; The cost of introducing the edge $\{u_{2n+1}, u_{2n+2}\}$ is $2W$; For remaining edges e, cost of adding e is still 0. The remainder of the proof is identical. □

F PROOF OF THEOREM 10

Proof. We reduce the PNM problem with asymmetric altruism from the KNAPSACK problem. The instance of KNAPSACK is the same as the proof of theorem 9.

We then construct an instance of PNM with asymmetric altruism. The relation network $G = (V, E)$ is defined as follows:

$$V = \{u_i : i \in [n + 2]\}$$
$$E = \{(u_i, u_j) : i, j \in [n + 2], i < j\}$$

Let $a = 1$ and the prosocial graph $H\,(V, E')$ be empty. Let the target profile x^* be the one where all players invest. It is equivalent to the profile that maximizes social welfare, which we will prove later. Now we define the functions $g_u(.)$ for all $u \in \mathcal{V}$. Set $g_{u_{n+1}}(x) = 0$ and $g_{u_{n+2}}(x) = x \cdot 2V$ for all $x \geqslant 0$. For all $i \in [n], g_{u_i}(x) = x \cdot v_i$ for all $x \geqslant 0$. Set $c_{u_{2n+1}} = V$ and for any other $i \in [n+2]$, $c_{u_i} = 0$. The cost of introducing the edge (u_{n+1}, u_i) is w_i for $i \in [n]$. For remaining edges e, cost of adding e is $2W$. Let the total budget be W. The reduction clearly can be constructed in polynomial time.

We then prove that in the construction above, the profile where all players invest maximizes social welfare. Since the relation graph is a clique, for all $i \in [n+2]$, if u_i invest, the positive change of social welfare from the contribution of $g_{u_{n+2}}$ is $2V$ and the cost is only V.

Then the target profile where the social welfare is maximized is equal to the target profile where all players invest.

Now we show that KNAPSACK Problem is a "YES" instance if and only if PNM with asymmetric altruism is a "YES" instance.

First, suppose that there is a set S of items that solves the KNAPSACK problem. Then, we introduce all the edges (u_{n+1}, u_i) for $i \in S$. Each node u_i will invest and the cost of introducing these edges will not exceed the budget W.

For the converse direction, let the PNM with asymmetric altruism be a "YES" instance. Set the set of edges introduced as S'. Let $S := \{i : (u_{n+1}, u_i) \in S'\}$. Hence if the subset S of items is chosen then we have $\sum_{i\in S} v_i \geqslant V$ and $\sum_{i\in S} w_i \leqslant W$. Hence the KNAPSACK problem is a "YES" instance.

The prove for PNM with symmetric altruism is similar. We reduce the problem to the same KNAPSACK problem, with the only difference being the definition of adding / removing edges: The cost of introducing the edge $\{u_{n+1}, u_i\}$ is w_i. For other edges e, cost of adding e is still $2W$. The remainder of the proof is identical. □

G PROOF OF THEOREM 11

Proof. To prove this claim, we shall prove that the modification solution for PNM problem with egalitarianism is just the solution for PNM problem with altruism.

For the target $\mathbf{x}^*$ in which all $x_v = 1$, we have:

$$U_v^{(F)}(1, \mathbf{x}_{-i}) = g_v(1+n_v) + a \min_{u'\in N_v} g_{u'}(x_{u'} + n_{u'}) - c_v \geq \tag{8}$$
$$g_v n_v + a \min_{u\in N_v} g_u x_u (x_u + n_u - 1) = U_v^{(F)}(0, \mathbf{x}_{-i}).$$

Let $w = \arg\min_{u\in N_v} g_u x_u (x_u + n_u - 1)$.

$$\begin{aligned}
U_v^{(A)}(1, \mathbf{x}_{-i}) &= g_v(1+n_v) + a \sum_{u\in N_v} g_u(x_u+n_u) - c_v \\
&= g_v(1+n_v) + a g_w(x_w+n_w) - c_v + a \sum_{u\in N_v/w} g_u(x_u+n_u) \\
&\geq g_v(1+n_v) + a \min_{u'\in N_v} g_{u'}(x_{u'}+n_{u'}) - c_v + a \sum_{u\in N_v/w} g_u(x_u+n_u) \\
&\geq g_v(n_v) + a g_w(x_w+n_w-1) + a \sum_{u\in N_v/w} g_u(x_u+n_u-1) \\
&= g_v(n_v) + a \sum_{u\in N_v} g_u x_u (x_u+n_u-1) = U_v^{(A)}(0, \mathbf{x}_{-i}).
\end{aligned}$$

The first inequality is correct because $g_w(x_w + n_w) \geq a \min_{u\in N_v} g_{u'}(x_{u'} + n_{u'})$. Eq. (8) ensures the second inequality. Thus, v won't deviate from playing 1 in PNM with altruism.

For the target $\mathbf{x}^*$ maximizing the social welfare, if $x_v = 0$, then

$$\begin{aligned}
&g_v(n_v) - g_v(1+n_v) + a \sum_{u\in N_v} [g_u(x_u+n_u) - g_u(x_u+n_u+1)] + c_v \\
&\geq g_v(n_v) - g_v(1+n_v) + \sum_{u\in N_v} [g_u(x_u+n_u) - g_u(x_u+n_u+1)] + c_v \\
&= SW(0, x_{-v}) - SW(1, x_{-v}) \geq 0,
\end{aligned}$$

where the first inequality is from the non-decreasing property of g, and the last inequality is from the assumption that $\mathbf{x}^*$ maximizes the social welfare. So

$$\begin{aligned}
U_v^{(A)}(0, \mathbf{x}_{-i}) &= g_u(n_u) + a \sum_{u\in N_v} g_u(x_u+n_u) \\
&\geq g_v(1+n_v) + a \sum_{u\in N_v} g_u(x_u+n_u+1) - c_v = U_v^{(A)}(1, \mathbf{x}_{-i}).
\end{aligned}$$

Obviously, v won't deviate from playing 0 in PNM with altruism. A similar analysis can be used to prove the case that $x_v = 1$. □

H PROOF OF THEOREM 12

Proof. Let us discuss the BNPG game on original network $G = H$ and suppose this game has a PSNE $\mathbf{x}$. In this PSNE, if $x_i = 0$,

$$U_i^{(C)}(0, \mathbf{x}_{-i}) = g_i(n_i) + \sum_{j\in N_i} g_j(x_j+n_j) \geq$$
$$U_i^{(C)}(1, \mathbf{x}_{-i}) = g_i(1+n_i) + \sum_{j\in N_i} g_j(x_j+n_j+1) - c_i. \tag{9}$$

Because $H = G$, we can not add edges into H for PNM problem. However, deleting edges cannot increase player i's willingness to invest, as the operations of deleting edges reduce N_i.

$$\begin{aligned}
&U_i^{(C)}(0, \mathbf{x}_{-i}) = g_i(n_i) + \sum_{j\in N_i'} g_j(x_j+n_j) \\
&\geq g_i(n_i) + \sum_{j\in N_i} g_j(x_j+n_j) - \sum_{j\in N_i/N_i'} g_j(x_j+n_j+1) \\
&\geq g_i(1+n_i) + \sum_{j\in N_i} g_j(x_j+n_j+1) - c_i - \sum_{j\in N_i/N_i'} g_j(x_j+n_j+1) \\
&= g_i(1+n_i) + \sum_{j\in N_i'} g_j(x_j+n_j+1) = U_i^{(C)}(1, \mathbf{x}_{-i}),
\end{aligned}$$

where the second inequality is from Eq.(9). By the similar analysis, we also can conclude that player i would not like to change her strategy 1 if some edges are deleted, for the case of $x_v = 1$.

Based on the above analysis, we claim that if target strategy profile $\mathbf{x}^* = (1, 1, \cdots, 1)$ changes to be a PSNE after deleting edges, then it must be a PSNE in original BNPG game. So, solving the PNM problem with symmetric collectivism is equivalent to checking whether $\mathbf{x}^* = (1, 1, \cdots, 1)$ is a PSNE in original BNPG game. □