On Green Sustainability of Resource Selection Games with Equitable Cost-Sharing

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ABSTRACT

As increasing concern for environmental sustainability urges to bring attention to green-aware multi-agent systems, we put forward a game-theoretic model in which agents compete for the usage of power-consuming resources and are charged a cost proportional to their fair share of the power consumption. Using the widely adopted cube-root rule for CMOS-based devices, our model becomes a congestion game in which two distinct parts coexist, namely, congestion games with polynomial latency functions and fair cost-sharing games. The interplay between these two components is governed by two resource-specific constants regulating the static and dynamic power consumption of each resource. Our findings show that, despite these games being highly inefficient in the general case (a super-constant price of stability), performance at equilibrium significantly improves (a constant price of anarchy) when the ratio between the static and dynamic power consumption of each resource remains bounded by a constant. This suggests that, in uncoordinated green-aware multi-agent systems, technology plays a fundamental role in shaping the efficiency of stable solutions.

CCS CONCEPTS

• **Theory of computation → Quality of equilibria**; **Algorithmic game theory**.

KEYWORDS

Green Sustainability; Nash Equilibria; Congestion Games; Price of Anarchy and Price of Stability

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1 INTRODUCTION

In our interconnected world, increasingly dependent on digital platforms and, at the same time, marked by growing concerns about environmental sustainability, a pressing issue demanding timely and efficient solution is the reduction of power consumption of Information Technology (IT) devices (personal computers, data centers, networks). It is highly anticipated, in fact, that their contribution to the annual electricity consumption in 2030 will exceed 10% of the total demand [34].

Numerous studies [20, 21, 37] have emphasized that the power consumption of IT devices, such as CMOS equipment, can be attributed to two main factors: a fixed demand related to the device activation and a variable, super-linear, demand related to speed. More precisely, the power consumption of a device r gets equal to

$$\alpha_r + \beta'_r {s_r}^{d'}, \tag{1}$$

where α_r and β'_r are two device-specific constants modelling, respectively, the *static* and *dynamic power consumption* of the device, s_r is its clock speed, and $d' \geq 1$ is a constant dependent of the specific technology, usually not exceeding 3, i.e., $1 \leq d' \leq 3$ [3, 37].

As IT devices are usually designed to run at their highest performance [36], one of the most effective approaches to reduce energy consumption is resorting to *speed scaling* to achieve *energy proportionality* [14]. This consists in dynamically maintaining speed of a device (e.g., a processor, a network link) proportional to its load, i.e., $s_r = c \cdot n_r$, so that also the energy consumption becomes proportional to the load. In such a setting, if one assumes that all users of a device r contribute equally to the load, the energy consumption becomes equal to

$$\alpha_r + \beta_r n_r^{d+1}, \tag{2}$$

where $d = d' - 1$ $(0 \leq d \leq 2)$[1], n_r is the number of users of r and $\beta_r = \beta'_r \cdot c^{d+1}$.

Inspired by this idea of green computing, we put forward a multi-agent system, termed *green-oriented resource selection game*,

[1] The reason why we switched from exponent d', with $d' \geq 1$, in (1) to exponent $d + 1$, with $d \geq 0$, in (2) will be clear when defining an agent's cost in (3).

populated by n non-cooperative selfish agents competing for the usage of a set of energy-consuming resources (IT devices). With the aim of keeping the overall energy consumption low, each agent is charged a cost equal to her contribution to the power consumption of each selected resource, so that the cost that each agent pays for using a resource r becomes

$$\frac{\alpha_r + \beta_r {n_r}^{d+1}}{n_r} = \frac{\alpha_r}{n_r} + \beta_r {n_r}^{d}. \tag{3}$$

According to this cost definition, green-oriented resource selection games become *congestion games* [39] in which two distinct parts coexist, namely, congestion games with polynomial latency functions [2, 8, 22, 24–26] and fair cost-sharing games [5, 16, 18, 32, 35]. As such, each green-oriented resource selection game is guaranteed to possess a pure Nash equilibrium.

1.1 Our Contribution

We investigate the (in)efficiency of pure Nash equilibria in green-oriented resource selection games through the notions of *price of anarchy* (PoA) [30] and *price of stability* (PoS) [5], measuring, respectively, the worst and the best possible ratio between the overall power consumption of a pure Nash equilibrium and that of an optimal solution.

We study the PoA and PoS under different assumptions on the resource-specific coefficients α_r and β_r. In the most general, unconstrained, case, it turns out that the PoA essentially matches the number of agents, as it is upper bounded by $n + o(n)$ and lower bounded by n. For the PoS, an upper bound of $(d + 1)H_n$ can be inferred from [5][2], which we complement with a lower bound of $\ln n \cdot \left(1 + \frac{2^d - 1}{2^{d+1}}\right)$ holding for every integer $d \geq 0$. We further show that this inefficiency remains even when assuming identical resources, that is, $\alpha_r = \alpha_{r'}$ and $\beta_r = \beta_{r'}$ for any two resources r and r' and even under network games, where resources are edges in a graph and each agent wants to connect two prescribed agent-specific vertices.

These lower bounds, however, rely on resources for which the ratio between the static and dynamic power consumption is unbounded. For such a reason, and in the hope of finding game classes with better performance, we also address the case in which, for each resource r, it holds that $\alpha_r \leq \theta\beta_r$, for some constant $\theta \geq 1$, that is, the ratio between the static and dynamic power consumption of each resource is upper bounded by a constant. As our main technical contribution, we show that, in this case, even the PoA becomes constant, as it only depends on constants d and θ. For the PoS, instead, we show a lower bound larger than one, meaning that no PNE can guarantee optimality, even in simple network games played on a parallel-link graph. Finally, we provide exact bounds for the PoA when the static and dynamic power consumption of each resource is equal and d is an integer, that is, $d \in \{0, 1, 2\}$.

Our findings shed light on how technological aspects may impact on the level of energy consumption yielded by stable solutions in non-cooperative multi-agent systems. In particular, when dealing with devices characterized by a huge diversity between static and dynamic power consumption, significantly poor solutions may emerge, highlighting the need for some form of control or coordination to achieve better final outcomes.

[2]Although in [5] it is mainly considered the cost-sharing case (i.e., the case in which the coefficient of the dynamic cost is set to 0), a combined model of costs and delays, which coincides with the one we analyze in this paper, is also addressed.

1.2 Related Work

The problem of energy-consumption minimization has been widely addressed in the literature, with a special focus on multi-processor scheduling and network routing. Energy efficient workload balance on data centers, realizing good trade-offs between energy and performance, has been studied in [6, 10–13, 27, 28, 33, 40], while the problem of determining communication paths and link speeds to meet Quality of Service (QoS) requirements with minimal energy usage has been considered in [3, 4, 7, 9, 23, 29, 31, 38]. These problems, however, have been mainly tackled assuming the presence of a centralized authority capable of computing and implementing an optimal solution (see [37] for a survey).

The game investigated in this paper can be modeled as a resource selection game in which the cost that each agent pays on each used resource is given by a combination of two terms: one that decreases with the number of users and one that polynomially increases with it. Games with the decreasing term only fall in the class of fair cost-sharing games, while games with the increasing term fall in the class of polynomial congestion games.

Fair cost-sharing games are often studied under the hypothesis that resources are edges in a graph and each agent has to buy a path connecting two prescribed agent-specific terminals (*network design games*). Two interesting subclasses of these games are *multicast games*, in which all agents share a common source terminal, and *broadcast games*, in which the common source has to be connected to all other vertices in the graph. For fair cost-sharing games with n agents, while the PoA is known to be exactly n, an upper bound of $H_n := \sum_{i=1}^{n} 1/i = O(\log n)$ on the PoS has been proven in [5]. The upper bound on the PoS has been shown to be tight even for broadcast games played on directed graphs. For the undirected case, upper bounds of $O(1)$, $O(\log n/\log\log n)$ and $H_{n/2}$ have been given in [18] for broadcast games, in [32] for multicast games, and in [35] for network design games, respectively. The best-known lower bounds, determined in [16], are 1.818 for broadcast games, 1.862 for multicast games and 2.245 for network design games.

For polynomial congestion games of maximum degree d, the PoA is a constant depending on d and growing asymptotically as $(d/\log d)^{d+o(d)}$ [2, 8, 26], while the PoS has been characterized in [22, 24, 25].

2 MODEL

For any fixed value $d \geq 0$,[3] a *green-oriented resource selection game* $G_d = (N, R, (S_i)_{i\in N}, (\alpha_r, \beta_r)_{r\in R})$ is defined by a set of n agents $N := \{1, \dots, n\}$, a set of resources R, a set of strategies $S_i \subseteq 2^R \setminus \emptyset$ for each agent $i \in N$ (notice that a strategy is given by a subset of resources) and two positive real coefficients α_r and β_r for each resource $r \in R$, governing the static and dynamic power consumption, respectively: for this reason, we will refer to these coefficients as the *static coefficient* and the *dynamic coefficient*, respectively.

[3]Although our model is motivated by applications that assume $0 \leq d \leq 2$, for theoretical purposes and to accommodate possible future technologies, we do not impose any upper bound on the value of d in our investigations.

A strategy profile $\boldsymbol{s} = (s_1, \ldots, s_n)$ is a vector of strategies, one for each agent. Let us denote with $n_r(\boldsymbol{s}) = |\{i \in N : r \in s_i\}|$ the number of users of resource r in $\boldsymbol{s}$, also known as the *congestion* of r in $\boldsymbol{s}$. Applying (2), we have that the overall power consumption induced by the agents' choices in $\boldsymbol{s}$ equals

$$\mathsf{PC}(\boldsymbol{s}) = \sum_{r \in R: n_r(\boldsymbol{s})>0} \left(\alpha_r + \beta_r n_r(\boldsymbol{s})^{d+1}\right).$$

Then, applying (3), we get that the cost that agent i pays in strategy profile $\boldsymbol{s}$ is defined as

$$\begin{aligned} cost_i(\boldsymbol{s}) &= \sum_{r \in s_i} \frac{1}{n_r(\boldsymbol{s})} \left(\alpha_r + \beta_r n_r(\boldsymbol{s})^{d+1}\right) \\ &= \sum_{r \in s_i} \left(\frac{\alpha_r}{n_r(\boldsymbol{s})} + \beta_r n_r(\boldsymbol{s})^d\right), \end{aligned}$$

that is, agents equally share their contribution to the overall power consumption. Observe that $\mathsf{PC}(\boldsymbol{s}) = \sum_{i \in N} cost_i(\boldsymbol{s})$.

A strategy profile $\boldsymbol{s}$ is a pure Nash equilibrium (PNE) if, for every agent $i \in N$ and every strategy $s \in S_i$, $cost_i(\boldsymbol{s}) \leq cost_i(\boldsymbol{s}_{-i}, s)$, where $\boldsymbol{s}_{-i}, s$ denotes the strategy profile obtained from $\boldsymbol{s}$ when i deviates to strategy s. Thus, in a PNE, no agent can lower her cost by unilaterally deviating to a different strategy. We shall denote with $\mathsf{PNE}(G_d)$ the set of PNE of game G_d.

Given a game G_d for which $\mathsf{PNE}(G_d) \neq \emptyset$, the *price of anarchy* of G_d is defined as $\mathsf{PoA}(G_d) = \max_{\boldsymbol{s} \in \mathsf{PNE}(G_d)} \frac{\mathsf{PC}(\boldsymbol{s})}{\mathsf{PC}(\boldsymbol{s}^*)}$, where $\boldsymbol{s}^*$ is the strategy profile minimizing the overall power consumption, also called *social optimum.* Conversely, the *price of stability* of G_d is defined as $\mathsf{PoS}(G_d) = \min_{\boldsymbol{s} \in \mathsf{PNE}(G_d)} \frac{\mathsf{PC}(\boldsymbol{s})}{\mathsf{PC}(\boldsymbol{s}^*)}$. So, the PoA and the PoS provide a pessimistic and optimistic measure, respectively, of the (in)efficiency of PNE. The PoA of a class of games C is defined as $\mathsf{PoA}(C) = \sup_{G_d \in C} \mathsf{PoA}(G_d)$ and the PoS as $\mathsf{PoS}(C) = \sup_{G_d \in C} \mathsf{PoS}(G_d)$.

We shall denote with $\mathcal{G}_d$ the set of all green-oriented resource selection games defined by fixing the value of $d \geq 0$. Then, with respect to the relations between the resource coefficients, we identify the following subclasses: *games with identical static coefficients*, in which $\alpha_r = \alpha_{r'}$ for each $r, r' \in R$; *games with identical dynamic coefficients*, in which $\beta_r = \beta_{r'}$ for each $r, r' \in R$; *games with identical resources*, in which $\alpha_r = \alpha_{r'}$ and $\beta_r = \beta_{r'}$ for each $r, r' \in R$; *games with θ-almost identical technologies* in which, for a given constant $\theta > 0$, $\alpha_r \leq \theta\beta_r$ for each $r \in R$; *games with identical technologies*, in which $\alpha_r = \beta_r$ for each $r \in R$; *games with identical resources and identical technologies*, in which $\alpha_r = \alpha_{r'} = \beta_r = \beta_{r'}$ for each $r, r' \in R$. We also say that a resource selection game is a *network game* if R is the set of edges of a graph and, for every agent i, S_i is the set of all paths connecting two prescribed agent-specific vertices: a source and a destination. A network game is a *parallel-link game* if all agents share the same source-destination pair and all paths are made of a single edge; however, not all paths may be available to an agent.

We conclude this section, by highlighting some fundamental connections that green-oriented resource selection games share with other well-studied classes of games. A congestion game $CG = (N, R, (S_i)_{i \in N}, (\ell_r)_{r \in R})$ is defined by a set of n agents $N := \{1, \ldots, n\}$, a set of resources R, a set of strategies $S_i \subseteq 2^R \setminus \emptyset$ for each agent $i \in N$ and a latency function $\ell_r : \mathbb{N} \mapsto \mathbb{R}$ for each resource $r \in R$. The cost of agent i in a strategy profile $\boldsymbol{s} = (s_1, \ldots, s_n)$ is defined as $cost_i(\boldsymbol{s}) = \sum_{r \in s_i} \ell_r(n_r(\boldsymbol{s}))$. Rosenthal's Theorem [39] states that any congestion game has the FIP property, that is, starting from any initial strategy profile, any sequence of better-response dynamics converges to a PNE after finitely many steps. This is proved by showing that function $\Phi(\boldsymbol{s}) = \sum_{r \in R} \sum_{j=1}^{n_r(\boldsymbol{s})} \ell_r(j)$ is an exact potential function, that is, $\Phi(\boldsymbol{s}_{-i}, s) - \Phi(\boldsymbol{s}) = cost_i(\boldsymbol{s}_{-i}, s) - cost_i(\boldsymbol{s})$ for each strategy profile $\boldsymbol{s}$, agent $i \in N$ and strategy $s \in S_i$.

The particular class of congestion games in which $\ell_r(x) = \beta_r x^d$ for each resource $r \in R$ is called *polynomial congestion games with degree d*, while the class in which $\ell_r(x) = \frac{\alpha_r}{x}$ for each resource $r \in R$ is called *fair cost-sharing games*. It is known that computing a PNE in both classes in PLS-complete [1, 17, 41].

By setting $\ell_r(x) = \frac{\alpha_r}{x} + \beta_r x^d$ for any $r \in R$, it turns out that any green-oriented resource selection game is a congestion game by definition. Thus, from Rosenthal's Theorem, we derive that any green-oriented resource selection game has the FIP property, and so, it admits at least one PNE. However, PLS-completeness of computing a PNE easily comes from PLS-completeness of both polynomial congestion games and fair cost-sharing games.

3 GENERAL GAMES

In this section, we analyse games without any type of restrictions on the adopted technologies, that is, class $\mathcal{G}_d$. One may observe that, as by letting the static coefficient α_r go to zero for each $r \in R$, the fair cost-sharing component of of green-oriented resource selection game becomes predominant, the lower bounds known for this class should carry over with some minor adaptations. Recall that, for fair cost-sharing games, the PoA is n and the PoS is H_n. Here, we show that, while the PoA remains essentially the same, the price of stability slightly worsens as d increases.

Before showing our theorems, we introduce some helpful additional notation. For a strategy profile $\boldsymbol{s}$ and a resource $r \in R$, we denote by $\mathbb{I}_r(\boldsymbol{s})$ the indicator function that equals 1 if $n_r(\boldsymbol{s}) > 0$ and equals 0 otherwise.

We start with the upper bound on the PoA.

Theorem 3.1. *For any $d \geq 0$, $\mathsf{PoA}(\mathcal{G}_d) \leq n$ if $d \leq 1$ and $\mathsf{PoA}(\mathcal{G}_d) = n + o(n)$ otherwise.*

Proof. To show the bounds, we make use of the primal-dual method [15, 19]. For a fixed game $G_d \in \mathcal{G}_d$, let $\boldsymbol{s}$ be a PNE and $\boldsymbol{s}^*$ be the social optimum. For the sake of conciseness, we set $k_r := n_r(\boldsymbol{s})$ and $o_r := n_r(\boldsymbol{s}^*)$. According to the primal-dual method, we need to formulate the problem of maximizing the power consumption of $\boldsymbol{s}$, subject to the constraint that the power consumption of $\boldsymbol{s}^*$ is normalized to one and to any constraint we can derive from the fact that $\boldsymbol{s}$ is a PNE. Then, any feasible solution to the dual of this problem will provide an upper bound on the PoA.

Consider the inequality $cost_i(\boldsymbol{s}) - cost_i(\boldsymbol{s}_{-i}, s_i^*) \leq 0$ for each $i \in N$. In our setting, this becomes

$$\sum_{r \in s_i} \left(\beta_r k_r^d + \frac{\alpha_r}{k_r}\right) - \sum_{r \in s_i^*} \left(\beta_r n_r(\boldsymbol{s}_{-i}, s_i^*)^d + \frac{\alpha_r}{n_r(\boldsymbol{s}_{-i}, s_i^*)}\right) \leq 0.$$

Define $k'_r := \min\{k_r + 1, n\}$, $k''_r := \max\{k_r, 1\}$ and observe that $n_r(\boldsymbol{s}_{-i}, s_i^*) \leq k'_r$ and $n_r(\boldsymbol{s}_{-i}, s_i^*) \geq k''_r$. As the dynamic power consumption is increasing in the the congestion and the static power consumption is decreasing in the congestion, the above inequality keeps holding if we replace the first occurrence of $n_r(\mathbf{s}_{-i}, s_i^*)$ with k'_r and its second occurrence with k''_r, so as to obtain inequality

$$\sum_{r\in s_i}\left(\beta_r k_r^d + \frac{\alpha_r}{k_r}\right) - \sum_{r\in s_i^*}\left(\beta_r (k'_r)^d + \frac{\alpha_r}{k''_r}\right) \le 0.$$

By summing previous inequality for each $i \in N$ and using $|\{i \in N : r \in s_i\}| = k_r$ and $|\{i \in N : r \in s_i^*\}| = o_r$, we obtain the inequality

$$\sum_{r\in R}\beta_r\left(k_r^{d+1} - o_r(k'_r)^d\right) + \sum_{r\in R}\alpha_r\left(\mathbb{I}_r(\mathbf{s}) - \frac{o_r}{k''_r}\right) \le 0.$$

Thus, the primal program is the following.

$$\begin{aligned}
\max \quad & \sum_{r\in R}\left(\beta_r k_r^{d+1} + \alpha_r \mathbb{I}_r(\mathbf{s})\right)\\
s.t. \quad & \sum_{r\in R}\beta_r\left(k_r^{d+1} - o_r(k'_r)^d\right)\\
& +\sum_{r\in R}\alpha_r\left(\mathbb{I}_r(\mathbf{s}) - \frac{o_r}{k''_r}\right) \le 0\\
& \sum_{r\in R}\left(\beta_r o_r^{d+1} + \alpha_r \mathbb{I}_r(\mathbf{s}^*)\right) = 1\\
& \alpha_r, \beta_r \ge 0 \qquad r \in R.
\end{aligned}$$

The dual program, obtained by associating variables x and y with the first and second constrain, respectively, is the following.

$$\begin{aligned}
\min \quad & \gamma\\
s.t. \quad & x\left(k_r^{d+1} - o_r(k'_r)^d\right) + \gamma o_r^{d+1} \ge k_r^{d+1} \quad & r \in R\\
& x\left(\mathbb{I}_r(\mathbf{s}) - \frac{o_r}{k''_r}\right) + \gamma \mathbb{I}_r(\mathbf{s}^*) \ge \mathbb{I}_r(\mathbf{s}) \quad & r \in R\\
& x \ge 0.
\end{aligned}$$

As any feasible solution for the dual is an upper bound to the optimal solution for the primal, we are left to show that there are suitable feasible dual solutions yielding the claimed bounds.

For $d \in [0, 1]$, set $x = 1$ and $\gamma = n$. The first constraint becomes $no_r^{d+1} \ge o_r(k'_r)^d$, which is always satisfied. The second constraint becomes $n\mathbb{I}_r(\mathbf{s}^*) \ge \frac{o_r}{k''_r}$ and is always satisfied.

For $d > 1$, set $x = 1+f(d,n)$ and $\gamma = n+nf(d,n)+g(d,n)$, where $f := f(d,n) = \frac{1}{\sqrt[2d]{n}}$ and $g := g(d,n) = 2^{d(d+2)}\sqrt{n}$. Observe that, being d an arbitrary, but fixed, constant, $nf + g = o(n)$. So, if we prove that the pair (x, γ) yields a feasible dual solution, the proof is complete.

The second constraint is satisfied as $x > 1, \gamma > nx$ and $\frac{o_r}{k''_r} \le n$. So, let us focus on the first constraint. By substituting and rearranging, we obtain the inequality

$$(n + nf + g)o_r^{d+1} - (1 + f)o_r(k'_r)^d + fk_r^{d+1} \ge 0. \tag{4}$$

If $k_r = 0$, which yields $k'_r = 1$, the inequality clearly holds. So, in the sequel, we assume $k_r \ge 1$, which implies that $k'_r \le 2k_r$. The derivative of the left-hand side with respect to o_r has a unique minimum corresponding to $o_r^* = k'_r\theta$, with $\theta := \sqrt[d]{\frac{1+f}{(d+1)(n+fn+g)}}$. Substituting and rearranging, we obtain that (4) is satisfied if and only if $fk_r^{d+1} \ge \frac{d(1+f)\theta(k'_r)^{d+1}}{d+1}$ is satisfied. As $f \le 1$ and $d > 1$, which yield $\theta \le \sqrt[d]{\frac{1}{g}}$, we derive

$$\begin{aligned}
\frac{d(1+f)\theta(k'_r)^{d+1}}{d+1} &\le \frac{2(k'_r)^{d+1}}{\sqrt[d]{g}}\\
&= \frac{2(k'_r)^{d+1}}{2^{d+2}\sqrt[2d]{n}}\\
&\le \frac{2(2k_r)^{d+1}}{2^{d+2}\sqrt[2d]{n}}\\
&= fk_r^{d+1},
\end{aligned}$$

as desired. □

The lower bound of n which holds for fair cost-sharing games can be easily extended to apply to green-oriented network games with identical static coefficients and, with an additive loss of 1, to even network games with identical resources.

Theorem 3.2. *For any $d \ge 0$, $\mathrm{PoA}(\mathcal{G}_d) \ge n$, even for network games with identical static coefficients and $\mathrm{PoA}(\mathcal{G}_d) \ge n-1$, even for network games with identical resources.*

For the PoS, an upper bound of $(d + 1)H_n$ comes from [5]. Although we are not able to show a matching lower bound for this metric, which is an open problem since 2004, we give two different results that allow us to claim interesting conclusions on the PoS. First, we give an H_n lower bound which holds even for network games with identical resources; secondly, we give an improved lower bound for the general case, showing that the PoS grows slightly larger than H_n and tends to $\frac{3}{2}H_n$ as d increases. This draws a separation between the behaviours of the PoA and the PoS of green-oriented resource selection games, when compared with the same metrics for fair cost-sharing games. In fact, while for every value of d, the PoA of green-oriented resource selection games is the same as that of fair cost-sharing games plus an asymptotically smaller term, the PoS increases by a non-negligible quantity when moving from fair cost-sharing games to green-oriented resource selection games.

In the same spirit of Theorem 3.2, also the lower bound of H_n holding for fair cost-sharing games can be extended to apply to green-oriented network games with identical resources, this time without any degradation.

Theorem 3.3. *For any $d \ge 0$, $\mathrm{PoS}(\mathcal{G}_d) \ge H_n$, even for network games with identical resources.*

Theorem 3.4. *For any integer $d \ge 0$ and any $\delta > 0$, there exists a game $\mathcal{G}_d$ such that $\mathrm{PoS}(\mathcal{G}_d) \ge \ln n \cdot \left(1 + \frac{2^d-1}{2^{d+1}}\right) - \delta$.*

Proof. For any fixed value of d and any integer g, let $\ell = 2^d$, $n = \ell^{2g}$ and consider game G_d with the last $n-1$ agents divided into $g = \log_{\ell^2} n$ groups, i.e., $N = \{1\} \cup N_0 \cup \ldots \cup N_{g-1}$, and $R = \{r_1\} \cup \bar{R}$, where $\bar{R} = \{r_2, \ldots, r_n\}$. Let $\alpha_{r_1} = 1$ and $\beta_{r_1} = \epsilon$; moreover, for every $j = 2, \ldots, n$, let $\alpha_{r_j} = \frac{\epsilon}{2}$ and $\beta_{r_j} = \frac{1}{j}$. The value of ϵ will be defined later. For any $k = 0, \ldots, g-1$, $|N_k| = \frac{\ell^2-1}{\ell^{2k+2}} \cdot n$ and $N_k = \{n - n \cdot \frac{\ell^{2k}-1}{\ell^{2k}}, n - n \cdot \frac{\ell^{2k}-1}{\ell^{2k}} - 1, \ldots, n - n \cdot \frac{\ell^{2k+2}-1}{\ell^{2k+2}}\}$. Every agent $i = 2, \ldots, n$ has two possible strategies, namely the *first* strategy and the *second* strategy of agent i, both composed by a unique resource:

the first strategy of agent i is $\{r_1\}$, and the second strategy of agent i is $\{r_{f(i)}\}$ with $r_{f(i)} \in \bar{R}$, where f is a function mapping every agent $i = 2, \ldots, n$ to a resource in $\bar{R}$. Agent 1 has only its first strategy $\{r_1\}$.

In order to define the second strategy of the agents, for any $k = 0, \ldots, g-1$, we divide group N_k in two subgroups: a subgroup composed by the first $\frac{\ell}{\ell+1}|N_k|$ agents and another subgroup composed by the remaining $\frac{1}{\ell+1}|N_k|$ agents. For $i = n - n \cdot \frac{\ell^{2k}-1}{\ell^{2k}}, \ldots, n - n \cdot \frac{\ell^{2k}-1}{\ell^{2k}} - \frac{\ell}{\ell+1}|N_k| + 1$, $f(i) = i$, i.e., for any agent i in the first subgroup of N_k, $\{r_i\}$ is assigned as her second strategy. For any $k = 0, \ldots, g-1$, for $i = n - n \cdot \frac{\ell^{2k}-1}{\ell^{2k}} - \frac{\ell}{\ell+1}|N_k|, \ldots, n - n \cdot \frac{\ell^{2k+2}-1}{\ell^{2k+2}}$, $f(i) = \ell \cdot i$, i.e., for any agent i in the second subgroup of N_k, the same second strategy of agent $\ell \cdot i$ (belonging to the first subgroup of N_k) is assigned as her second strategy. We say that two agents having the same second strategy are *sibling*, with the elder sibling being the one identified by a greater index (and thus belonging to the first subgroup of her group) and the youngest sibling belonging to the second subgroup of the same group.

Consider strategy profiles s^1 and s^2 in which all agents select their first strategies and their second strategies, respectively.

On the one hand, since in s^1 only resource r_0 is used by all agents, it holds that $\text{PC}(s^1) = \alpha_{r_0} + n \cdot \beta_{r_0} n^d =\leq 1 + \epsilon n^{d+1}$. On the other hand, in order to compute $\text{PC}(s^2)$, we notice that $\text{PC}(s^2)$ is equal to the sum of all static costs c_0 of the used resources in $\bar{R}$ plus the sum of the dynamic costs that all agents $n, n-1, \ldots, 2$ experience, at the moment of their deviation, in the dynamics starting from $\text{PC}(s^1)$ and leading to $\text{PC}(s^2)$ by letting agents move in decreasing order (from agent n to agent 2), plus an extra cost c due to the fact that the the youngest siblings, when deviating, increase the dynamic cost of the respective oldest siblings, plus the cost of agent 1 remaining alone on resource r_1. We have that, for any agent i either not having siblings or being the oldest sibling, at the moment of the deviation, the dynamic cost on the second strategy r_i is $\frac{1}{i}$; for any agent i being the youngest sibling, at the moment of the deviation, the dynamic cost on the second strategy $r_{i \cdot \ell}$ is $\frac{2^d}{i \cdot \ell} = \frac{1}{i}$, because her oldest sibling is already using resource $r_{i \cdot \ell}$. Therefore, we have $\text{PC}(s^2) = c_0 + \sum_{i=2}^{n} \frac{1}{i} + c + 1 + \epsilon = c_0 + c + \epsilon + H_n \geq c + \ln n$.

In order to provide a lower bound to c, we sum over, for all i being the youngest sibling of $\ell \cdot i$, the extra cost $\frac{\ell-1}{\ell \cdot i}$ they add to agent $\ell \cdot i$. We obtain

$$
\begin{aligned}
c &= \sum_{k=0}^{g-1} \sum_{i=\frac{n}{\ell^{2k+2}}}^{\frac{n}{\ell^{2k+1}}-1} \frac{\ell-1}{\ell \cdot i} \\
&= \frac{\ell-1}{\ell} \sum_{k=0}^{g-1} \sum_{i=\frac{n}{\ell^{2k+2}}}^{\frac{n}{\ell^{2k+1}}-1} \frac{1}{i} \\
&= \frac{\ell-1}{\ell} \sum_{k=0}^{g-1} \left(\psi\left(\frac{n}{\ell^{2k+1}}\right) - \psi\left(\frac{n}{\ell^{2k+2}}\right) \right) \\
&\geq \frac{\ell-1}{\ell} \sum_{k=0}^{g-1} \left(\ln \frac{n}{\ell^{2k+1}} - \frac{\ell^{2k+1}}{n} - \ln \frac{n}{\ell^{2k+2}} + \frac{\ell^{2k+2}}{2n} \right) \\
&\geq \frac{\ell-1}{\ell} \sum_{k=0}^{g-1} \ln \ell = \frac{\ell-1}{\ell} \cdot g \cdot \ln \ell \\
&= \frac{\ell-1}{\ell} \cdot \log_{\ell^2} n \cdot \ln \ell = \ln n \cdot \frac{\ell-1}{2\ell},
\end{aligned}
$$

where the first inequality holds because the digamma function $\psi(x)$ is such that $\ln x - \frac{1}{x} \leq \psi(x) \leq \ln x - \frac{1}{2x}$ for any $x > 0$. Therefore, given any $\delta > 0$, it is possible to choose ϵ such that

$$
\frac{\text{PC}(s^2)}{\text{PC}(s^1)} \geq \ln n \cdot \left(1 + \frac{\ell-1}{2\ell}\right) - \delta = \ln n \cdot \left(1 + \frac{2^d-1}{2^{d+1}}\right) - \delta.
$$

In the following, we show that the only Nash equilibrium of G_d is s^2. To this aim, let s be any strategy profile different from s^2 and let s' the strategy profile obtained by s by swapping the strategies of two siblings i, i' with $i > i'$ if it happens that i and i' are selecting their first and second strategy in s, respectively. Clearly, $\text{PC}(s) = \text{PC}(s')$ and, since two siblings have the same strategy set, it holds that s is a Nash equilibrium if and only if s' is a Nash equilibrium. Notice that s' has the property that whenever only a sibling is using r_1, she is the youngest sibling. Let $A \neq \emptyset$ be the set of agents selecting their first strategy in s': we show that s' is not a Nash equilibrium because there exists an agent in A that can perform an improving move. Let $m = \max_{i \in A} i$. Notice that $n_{r_1}(s') \leq m$. We distinguish among the following disjoint cases:

- m has a sibling.
  - If m is the youngest sibling, we have that $cost_m(s') \geq \frac{1}{m} + \epsilon$, while the cost that agent m would pay for her second strategy, i.e., on resource $r_{m \cdot \ell}$ (notice that for an agent i with an older sibling it holds that $f(i) = i \cdot \ell$), is at most $\frac{2^d}{m \cdot \ell} + \frac{\epsilon}{2} = \frac{1}{m} + \frac{\epsilon}{2}$ because every resource in $\bar{R}$ can have congestion at most 2: s' is not a Nash equilibrium.
  - If m is the oldest sibling, by construction of s', the youngest sibling $\frac{m}{\ell}$ of m is in A. We have that $cost_m(s') \geq \frac{1}{m} + \epsilon$, while the cost that agent m would pay for her second strategy, i.e., on resource r_m (notice that for an agent i with a younger sibling it holds that $f(i) = i$), is at most $\frac{1}{m} + \frac{\epsilon}{2}$ because $\frac{m}{\ell} \in A$ implies $n_{r_m}(s') = 0$: s' is not a Nash equilibrium.
- m does not have any sibling. We have that $cost_m(s') \geq \frac{1}{m} + \epsilon$, while the cost that agent m would pay for her second strategy, i.e., on resource r_m (notice that for an agent i without siblings it holds that $f(i) = i$), is at most $\frac{1}{m} + \frac{\epsilon}{2}$ because every resource in $\bar{R}$ used by an agent without siblings can have congestion at most 1: s' is not a Nash equilibrium.

□

We stress that, although the lower bound shown in the above theorem might look slightly incremental with respect to the previously known one, it represents indeed the first improvement since 2004, when the conference version of [5] appeared.

4 GAMES WITH SIMILAR TECHNOLOGIES

Given the high inefficiency of PNE in general games, in this section, we investigate whether, with similar technologies, better performance at equilibria are possible.

We have already seen in the previous section, and in particular in Theorems 3.2 and 3.3, that even restricting to network games with identical resources does not help to improve the performance at PNE. Considering that the PoA and PoS of a class of games are worst-case measures among all games in the class, it may be the case that, on some instances, better performance can be achieved when assuming identical coefficients and even identical resources. However, the next theorem shows that this is not the case for games with either identical static coefficients or identical dynamic coefficients. Moreover, it also states that having identical resources does not improve the performance of games with identical technologies.

Theorem 4.1. *For each game $G_d \in \mathcal{G}_d$, there exist a game with identical static coefficients G'_d and a game with identical dynamic coefficients G''_d such that $\mathsf{PoA}(G'_d) = \mathsf{PoA}(G''_d) = \mathsf{PoA}(G_d)$ and $\mathsf{PoS}(G'_d) = \mathsf{PoS}(G''_d) = \mathsf{PoS}(G_d)$. Moreover, for each game with identical technologies G_d, there exists a game with identical resources and identical technologies G'_d such that $\mathsf{PoA}(G'_d) = \mathsf{PoA}(G_d)$ and $\mathsf{PoS}(G'_d) = \mathsf{PoS}(G_d)$.*

The next theorem is our main technical contribution and shows that, for any fixed $d \geq 0$ and constant $\theta > 0$, the PoA becomes constant in games with θ-almost identical technologies. We also provide specific upper bounds for any $d \geq 0$ and $\theta > 0$.

Theorem 4.2. *For any game with θ-almost identical technologies G_d, with $d \geq 0$ and $\theta > 0$, $\mathsf{PoA}(G_d)$ is constant. In particular, $\mathsf{PoA}(G_d) \leq (\Phi_d(\theta))^{d+1}$, where $\Phi_d(\theta) = O\left(d/\log(d) + \sqrt[d+1]{\theta}\right)$ is the unique solution t of equation $-t^{d+1} + (t+1)^d + \theta = 0$.*

Proof. For the upper bounds, we make again use of the primal-dual method. However, with respect to the formulation derived in the proof of Theorem 3.1, this time we can somehow relate the two coefficients for every resource $r \in R$, that is, we can set $\alpha_r := \theta_r \beta_r$ for some fixed parameter $\theta_r \in [0, \theta]$.

Thus, the primal program P becomes the following.

$$\begin{aligned}
\max \quad & \sum_{r\in R}\left(\beta_r k_r^{d+1} + \beta_r\theta_r \mathbb{I}_r(s)\right) \\
s.t. \quad & \sum_{r\in R}\beta_r\left(k_r^{d+1} - o_r(k'_r)^d\right) \\
& + \sum_{r\in R}\beta_r\left(\theta_r\mathbb{I}_r(s) - \frac{\theta_r o_r}{k''_r}\right) \leq 0 \\
& \sum_{r\in R}\left(\beta_r o_r^{d+1} + \beta_r\theta_r\mathbb{I}_r(s^*)\right) = 1 \\
& \beta_r \geq 0 \qquad r \in R.
\end{aligned}$$

The dual program PP is the following.

$$\begin{aligned}
\min \quad & \gamma \\
s.t. \quad & \gamma\left(o_r^{d+1} + \theta_r\mathbb{I}_r(s^*)\right) \geq k_r^{d+1} + \theta_r\mathbb{I}_r(s) \\
& +x\left(-k_r^{d+1} - \theta_r\mathbb{I}_r(s) + o_r(k'_r)^d + \tfrac{\theta_r o_r}{k''_r}\right) \qquad r \in R \\
& x \geq 0.
\end{aligned}$$

We have the following lemma.

Lemma 4.3. *Let (γ, x) be a pair of real values such that $x > 1$ and*

$$\gamma \geq t^{d+1} + x\left(-t^{d+1} + (t+1)^d + \theta\right) \quad \forall t \geq 0. \tag{5}$$

Then, (γ, x) is a feasible solution of the dual program DP.

Proof of Lemma 4.3. First of all, one can easily see that (γ, x) trivially satisfies the constraints of DP with $o_r = 0$. Thus, let us show the claim for $o_r > 0$. Assume by contradiction that (γ, x) does not satisfy a dual constraint associated with a resource r with $o_r > 0$, and let $t := k_r/o_r$. Recalling that $k'_r := \min\{k_r + 1, n\}$ and $k''_r := \max\{k_r, 1\}$, we have

$$\begin{aligned}
\gamma &< \frac{k_r^{d+1} + \theta_r\mathbb{I}_r(s) + x\left(-k_r^{d+1} - \theta_r\mathbb{I}_r(s) + o_r(k'_r)^d + \frac{\theta_r o_r}{k''_r}\right)}{\left(o_r^{d+1} + \theta_r\right)} \qquad (6)\\
&\leq \frac{k_r^{d+1} + x\left(-k_r^{d+1} + o_r(k'_r)^d + \frac{\theta_r o_r}{k''_r}\right)}{\left(o_r^{d+1} + \theta_r\right)} \\
&\leq \frac{k_r^{d+1} + x\left(-k_r^{d+1} + o_r(k'_r)^d + \frac{\theta_r o_r}{k''_r}\right)}{o_r^{d+1}} \\
&\leq \frac{k_r^{d+1} + x\left(-k_r^{d+1} + o_r(k_r + o_r)^d + \theta_r o_r^{d+1}\right)}{o_r^{d+1}} \qquad (7)\\
&= t^{d+1} + x\left(-t^{d+1} + (t+1)^d + \theta_r\right) \\
&\leq t^{d+1} + x\left(-t^{d+1} + (t+1)^d + \theta\right) \\
&\leq \gamma, \qquad (8)
\end{aligned}$$

where (6) holds since $(1 - x)\theta\mathbb{I}_r(s)$ is non-positive (as $x > 1$ by the hypothesis), (7) follows as $k'_r \leq k_r + 1$, $o_r \geq 1$, and $k''_r \geq 1$, and (8) holds by (5). Thus, we obtain $\gamma < \gamma$, that is, a contradiction. □

By the above lemma, we have that, if we find a pair (γ, x) with $x > 1$ that satisfies (5), we obtain the claim. To find such a pair, we will show some qualitative properties on the real numbers $t(x)$ that, for any $x > 1$, maximize the right-hand part of (5), denoted as $f_x(t)$. We first provide a preliminary technical lemma.

Lemma 4.4. *For any $a > 0, b > 0, c \geq 0$ and d in interval $[i - 1, i)$, the function $g(t) := g_{a,b,c,d}(t)$, defined on the set of non-negative reals as $-a \cdot t^{d+1} + b \cdot (t+1)^d + c$, admits a unique maximum point $t' := t'(a, b, c, d) > 0$ and a unique zero $t^* := t^*(a, b, c, d) > t'$ such that $g(t) > 0$ for any $t \in [0, t^*)$ and $g(t) < 0$ for any $t > t^*$.*

Proof. We will show the claim by induction on $i \in \mathbb{N}$. If $i = 1$, it means that $d \in [0, 1)$, and in such a case $g(t)$ is concave (as it can be seen as the weighted sum of the three functions $-t^{d+1}$, $(t + 1)^d$, c, which are concave for $d \in [0, 1)$). Since $g'(0) > 0$, concavity implies that g admits a unique maximum point $t' > 0$. Now, since $g(0) > 0$ and $\lim_{t\to\infty} g(t) = -\infty$, continuity of g (by the Zero Existence Theorem) implies the existence of a unique zero $t^* > t'$ of g. Furthermore, as there exists a unique maximum t' of function $g(t)$, we have that t^* is necessarily the unique zero of $g(t)$, so that $g(t) > 0$ for any $t \in [0, t^*)$ and $g(t) < 0$ for $t > t^*$.

Now, assume that the claim holds for some $i \in \mathbb{N}$, and let us show it for $i + 1$. Let $d \in [i, i + 1)$. We first study the properties of maximum points of $g(t)$ by means of their characterization via derivatives. We have that $g'(t) = -\tilde{a}t^{\tilde{d}+1} + \tilde{b} \cdot (t+1)^{\tilde{d}} + \tilde{c}$, with $\tilde{a} = (d + 1)a$, $\tilde{b} = db$, $\tilde{c} = 0$ and $\tilde{d} = d - 1 \in [i - 1, i)$. Thus, since $\tilde{d} \in [i - 1, i)$, we can apply the inductive hypothesis to $g'(t)$ with the considered values $\tilde{d}, \tilde{a}, \tilde{b}, \tilde{c}$, that is, there exists $\tilde{t^*} > 0$ such that

$g'(t) > 0$ for any $t \in [0, \tilde{t^*})$ and $g'(t) < 0$ for any $t > \tilde{t}$. This means that $g(t)$ is increasing for $t \in [0, \tilde{t^*})$ and decreasing for $t > \tilde{t^*}$, that is, $t' := \tilde{t^*}$ is the unique maximum point of g. As $g(0) = 0$, $\lim_{t\to\infty} g(t) = -\infty$, g is continuous, and g has a unique maximum point t', as in the base case these properties necessarily imply the existence of a unique zero $t^* > t'$ of $g(t)$ such that $g(t) > 0$ for any $t \in [0, t^*)$ and $g(t) < 0$ for $t > t^*$. This shows the inductive step, and then the claim of the lemma. □

For any fixed $x > 0$, we have that the constraints (5) can be seen as $\gamma \geq f_x(t)$ for any $t \geq 0$, where $f_x(t)$ is defined as $-at^{d+1} + b(t+1)^d + c$, with $a := x - 1 > 0$, $b := x > 0$ and $c = x\theta \geq 0$. Then, by Lemma 4.4, we have that $f_x(t)$ admits a unique maximum point, that is, the function $t(x)$ returning such a maximum point is well-defined for any $x > 1$.

Let $\Phi_d(\theta)$ be the unique solution t of equation $-t^{d+1} + (t+1)^d + \theta = 0$. We observe that existence and uniqueness of such a solution are well-defined, as $-t^{d+1} + (t+1)^d + \theta$ can be written as a function $g(t)$ satisfying the hypothesis of Lemma 4.4, and then it admits a unique zero $t^* > 0$.

Now, in the following lemma we will show the existence of a value $x > 1$ such that the unique maximum point $t(x)$ of f_x is exactly $\Phi_d(\theta)$.

Lemma 4.5. *There exists $x > 1$ such that $t(x) = \Phi_d(\theta)$.*

By using the value $x > 1$ such that $t(x) = \Phi_d(\theta)$ (determined in the above lemma) as a parameter of the function f_x and by setting $\gamma := (\Phi_d(\theta))^{d+1}$, we have that the following inequalities hold for any $t \geq 0$:

$$\begin{aligned} f_x(t) &\leq f_x(t(x)) && (9)\\ &= t(x)^{d+1} + x(-t(x)^{d+1} + (t(x)+1)^d + \theta)\\ &= t(x)^{d+1} && (10)\\ &= (\Phi_d(\theta))^{d+1}\\ &= \gamma, \end{aligned}$$

where (9) holds as $t(x)$ is the maximum point of $f_x(t)$, and (10) holds since $t(x) = \Phi_d(\theta)$ is the solution of equation $-t(x)^{d+1} + (t(x)+1)^d + \theta = 0$. Thus, we showed that there exists $x > 1$ such that pair $(\gamma = (\Phi_d(\theta))^{d+1}, x)$ is feasible for (5), and by Lemma 4.3, this shows the claim.

It remains to show that $\Phi_d(\theta)$ grows as $O\left(d/\log(d) + \sqrt[d+1]{\theta}\right)$. In the full version we prove that there exists a constant $c > 0$ such that the value $t := c \cdot \max\left\{d/\log(d), \sqrt[d+1]{\theta}\right\}$ satisfies $-t^{d+1} + (t+1)^d + \theta \leq 0$. Thus, as $\Phi_d(\theta)$ is the unique zero of equation $-t^{d+1} + (t+1)^d + \theta = 0$, by Lemma 4.4 we necessarily have that $\Phi_d(\theta) \leq c \cdot \max\left\{d/\log(d), \sqrt[d+1]{\theta}\right\} = O\left(d/\log(d) + \sqrt[d+1]{\theta}\right)$. □

The upper bounds showed in the previous theorem are, in general, not tight. In the following, we consider games with identical technologies when d assumes integral values in our reference interval $[0, 2]$.

Theorem 4.6. *For any game with identical technologies G_d, we have* $\mathrm{PoA}(G_d) \leq 2$ *if $d = 0$,* $\mathrm{PoA}(G_d) \leq 20/13$ *if $d = 1$ and* $\mathrm{PoA}(G_d) \leq 1381/290 \approx 4.762$ *if $d = 2$. Moreover, all bounds are tight.*

Proof. For the upper bounds, we make again use of the primal-dual method. However, with respect to the formulation derived in the proof of Theorem 3.1, this time we have the freedom to choose a unique coefficient for every resource, that we denote as α_r. Moreover, in order to achieve a tight bound, we shall need a more refined formulation.

Consider the inequality

$$\sum_{r\in s_i}\left(\beta_r k_r^d + \frac{\beta_r}{k_r}\right) - \sum_{r\in s_i^*}\left(\beta_r n_r(s_{-i}, s_i^*)^d + \frac{\beta_r}{n_r(s_{-i}, s_i^*)}\right) \leq 0,$$

modeling the fact that no agent can lower her cost by deviating to the strategy she plays in the social optimum. Considering that, for any resource $r \in s_i \cap s_i^*$, the contributions in the two summations are the same and that, for any resource $r \in s_i^* \setminus s_i$, it holds that $n_r(s_{-i}, s_i^*) = n_r(s) + 1 = k_r + 1$, we obtain

$$\sum_{r\in s_i\setminus s_i^*}\left(\beta_r k_r^d + \frac{\beta_r}{k_r}\right) - \sum_{r\in s_i^*\setminus s_i}\left(\beta_r (k_r+1)^d + \frac{\beta_r}{k_r+1}\right) \leq 0.$$

Define $\delta_r = |\{i \in N : r \in s_i \cap s_i^*\}|$ as the number of agents selecting resource r in both $\boldsymbol{s}$ and $\boldsymbol{s}^*$. Observe that, by definition, $\delta_r \leq \min\{k_r, o_r\}$. By summing previous inequality for each $i \in N$, we obtain the inequality

$$\sum_{r\in R}\beta_r\left((k_r-\delta_r)k_r^d - (o_r-\delta_r)(k_r+1)^d\right) + \sum_{r\in R}\beta_r\left(\frac{k_r-\delta_r}{k_r} - \frac{o_r-\delta_r}{k_r+1}\right) \leq 0,$$

with the interpretation that, for $k_r = 0$, which implies $\delta_r = 0$, ratio $\frac{k_r-\delta_r}{k_r} = \frac{0}{0}$ is set equal to 0.

Under these premises, the primal program becomes the following.

$$\begin{aligned} \max\quad & \sum_{r\in R}\left(\beta_r k_r^{d+1} + \beta_r \mathbb{I}_r(\boldsymbol{s})\right)\\ s.t.\quad & \sum_{r\in R}\beta_r\left((k_r-\delta_r)k_r^d - (o_r-\delta_r)(k_r+1)^d\right)\\ & + \sum_{r\in R}\beta_r\left(\frac{k_r-\delta_r}{k_r} - \frac{o_r-\delta_r}{k_r+1}\right) \leq 0\\ & \sum_{r\in R}\left(\beta_r o_r^{d+1} + \beta_r \mathbb{I}_r(\boldsymbol{s}^*)\right) = 1\\ & \beta_r \geq 0 \qquad r \in R. \end{aligned}$$

The dual program is the following.

$$\begin{aligned} \min\quad & \gamma\\ s.t.\quad & x\left((k_r-\delta_r)k_r^d - (o_r-\delta_r)(k_r+1)^d + \tfrac{k_r-\delta_r}{k_r} - \tfrac{o_r-\delta_r}{k_r+1}\right)\\ & + \gamma\left(o_r^{d+1} + \mathbb{I}_r(\boldsymbol{s}^*)\right) \geq k_r^{d+1} + \mathbb{I}_r(\boldsymbol{s}) \qquad r \in R\\ & x \geq 0. \end{aligned}$$

For $d = 0$, set $x = 1$ and $\gamma = 2$. By substituting in the unique constraint, we get inequality $2o_r + 2\mathbb{I}_r(\boldsymbol{s}^*) + \frac{k_r-\delta_r}{k_r} \geq \mathbb{I}_r(\boldsymbol{s}) + o_r + \frac{o_r-\delta_r}{k_r+1}$, which is always satisfied. In fact, if $o_r \geq 1$, the left-hand side is at least $2o_r + 2$, while the right-hand side at most $2o_r + 1$;

if $o_r = 0$, which implies $\delta_r = 0$, the left-hand side is $\mathbb{I}_r(s)$ which equals the right-hand side.

For $d = 1$, set and $x = \gamma = 20/13$. Assume first that $k_r = 0$, which yields $\mathbb{I}_r(s) = 0$ and $\delta_r = 0$. By substituting in the unique constraint, we get inequality $\frac{20}{13}\left(o_r^2 - 2o_r + \mathbb{I}_r(s^*)\right) \geq 0$, which is always satisfied. Now assume that $o_r = 0$, which yields $\mathbb{I}_r(s^*) = 0$ and $\delta_r = 0$. By substituting, we get inequality $\frac{7}{13}(k_r^2 + \mathbb{I}_r(s)) \geq 0$, which is always satisfied. So, let us focus on the case in which $k_r, o_r \geq 1$, which yields $\mathbb{I}_r(s) = \mathbb{I}_r(s^*) = 1$. By substituting, we get that the dual constraint is satisfied if and only if inequality $7k_r^4 + k_r^3(7 - 20o_r) + k_r(k_r + 1)(20o_r^2 - 40o_r + 20\delta_r + 27) - 20\delta_r \geq 0$ holds true. The left-hand side of this inequality is increasing in δ_r, so we show that the inequality remains true even for $\delta_r = 0$, which yields $7k_r^4 + k_r^3(7 - 20o_r) + k_r(k_r + 1)(20o_r^2 - 40o_r + 27) \geq 0$. The left-hand side of this inequality is minimized when $o_r = \frac{k_r^2+2k_r+2}{2(k_r+1)}$. By substituting, we derive that the dual constraint is satisfied if inequality $2k_r^4 - 8k_r^3 + 2k_r^2 + 12k_r + \frac{5}{k_r+1} - 5 \geq 0$ holds true. This holds true for any value of $k_r \neq 2, 3$. By inspecting the dual constraint with these two specific values of k_r and $\delta_r = 0$, we get inequalities $3o_r^2 - 10o_r + \frac{33}{4} \geq 0$ and $4o_r^2 - 17o_r + 18 \geq 0$ which are satisfied by any integral value of o_r.

For $d = 2$, set and $x = 228/145$ and $\gamma = 1381/290$. Assume first that $k_r = 0$, which yields $\mathbb{I}_r(s) = 0$ and $\delta_r = 0$. By substituting in the unique constraint, we get inequality $1381\left(o_r^3 + \mathbb{I}_r(s^*)\right) \geq 912o_r$, which is always satisfied. Now assume that $o_r = 0$, which yields $\mathbb{I}_r(s^*) = 0$ and $\delta_r = 0$. By substituting, we get inequality $\frac{83}{145}(k_r^3 + \mathbb{I}_r(s)) \geq 0$, which is always satisfied. So, let us focus on the case in which $k_r, o_r \geq 1$, which yields $\mathbb{I}_r(s) = \mathbb{I}_r(s^*) = 1$. By substituting, we get that the dual constraint is satisfied if and only if inequality $166k_r^5 + 2k_r^4(83 - 228o_r) + 456k_r^3(2\delta_r - 3o_r) + k_r^2(1381o_r^3 - 1368o_r + 1368\delta_r + 1547) + k_r(1381o_r^3 - 912o_r + 456\delta_r + 1547) - 456\delta_r \geq 0$ holds true. The left-hand side of this inequality is increasing in δ_r, so we show that the inequality remains true even for $\delta_r = 0$, which yields $166k_r^4 + 2k_r^3(83 - 228o_r) - 1368k_r^2o_r + k_r(1381o_r^3 - 1368o_r + 1547) + 1381o_r^3 - 912o_r + 1547 \geq 0$. The left-hand side of this inequality is minimized when $o_r = \frac{2\sqrt{\frac{52478(k_r^3+3k_r^2+3k_r+2)}{k_r+1}}}{1381}$. By substituting, we derive that the dual constraint is satisfied if inequality $\sqrt{1381}(166k_r^4 + 166k_r^3 + 1547k_r + 1547k_r) - 608\sqrt{38}(k_r^3 + 3k_r^2 + 3k_r + 2)\sqrt{\frac{k_r^3+3k_r^2+3k_r+2}{k_r+1}} \geq 0$ holds true. Assume $k_r \geq 5$. As $\frac{k_r^3+3k_r^2+3k_r+2}{k_r+1} \leq \frac{29}{20}k_r^2$ for each $k_r \geq 5$, the claim follows for any $k_r \geq 5$ if inequality $\sqrt{1381}(166k_r^4 + 166k_r^3 + 1547k_r + 1547k_r) - 608\sqrt{38}(k_r^3 + 3k_r^2 + 3k_r + 2)\sqrt{\frac{29}{20}}k_r \geq 0$ holds true, which is indeed the case. So, only the cases of $1 \leq k_r \leq 4$ are left over. By inspecting the dual constraint with these four specific values of k_r and $\delta_r = 0$, we get inequalities $1381o_r^3 - 2052o_r + 1713 \geq 0$, $1381o_r^3 - 4256o_r + 2875 \geq 0$, $1381o_r^3 - 7410o_r + 6029 \geq 0$ and $6905o_r^3 - 57456o_r + 60855 \geq 0$ which are all satisfied by any integral value of o_r.

In the full version we show that the above upper bounds are tight. □

Finally, we prove that similar technologies cannot guarantee optimality at equilibrium, as the price of stability is bounded away from one even in games with identical technologies played on parallel-link graphs.

Theorem 4.7. *For any $d \geq 0$, there exists a game with identical technologies G_d such that $\mathrm{PoS}(G_d) > 1$ even when restricting to parallel-link games.*

5 CONCLUSIONS

Inspired by power management techniques in green computing, we put forward a game theoretic model in which a set of non-cooperative selfish agents, competing for the usage of energy consuming resources, are charged a cost depending on the power consumption they demand to the system. We have shown that good performance can be achieved only when the maximum ratio between the coefficients regulating the static and dynamic power consumption of a resource is bounded by a constant. It is worth noticing that, given the non-negligibility of static coefficients in modern real world hardware (e.g., due to the cost for maintaining the device on, even if it is idle - see lines 8-10 of Section 3 in [36]), the ratio between the dynamic and static coefficients is naturally bounded by a constant in common real world scenarios.

We leave several open problems, such as closing the gap between upper and lower bounds on the PoS for general games and obtaining a precise characterization of the PoA, as well as some non-trivial upper bounds on the PoS, for games with θ-almost identical technologies. Finally, our work is a first attempt to include green-awareness within multi-agent systems populated by rational selfish agents. This approach can be generalized along several directions and adapted in a variety of distributed scenarios, and we believe that this may spur further research in the field. For instance, it would be interesting to consider a setting with heterogeneous users, where each user contributes to the energy consumption in a proportional way, according to an individual weight. Furthermore, we note that our results can be slightly modified so that they continue to hold even if the dynamic power consumption is a general polynomial of maximum degree $d + 1$[4].

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