Metric Distortion Under Public-Spirited Voting

Extended Abstract

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ABSTRACT

We investigate the impact of public-spirited voting on the distortion in the metric framework. We employ the public-spirited model proposed by Flanigan *et al.* [3] (EC'23) to model the public-spirited behavior of the agents and evaluate the distortion of different voting rules, including Plurality, Borda, Copeland, Veto, k-approval, and PluralityVeto. We establish a lower bound for any voting rule operating within the metric framework with public-spirited voters. Additionally, we present lower and upper bounds on the distortion associated with these voting rules within the public-spirited model. Among these voting rules, we show that, in the case of public-spirited voting where all voters exhibit identical behavior, the distortion of PluralityVeto matches the general lower bound.

KEYWORDS

Distortion; Metric; Voting; Election; Public-spirited

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1 INTRODUCTION

In social choice theory, an election is a mechanism used to transform individual preferences into a collective decision. Generally, an election involves a group of n voters and a set of m alternatives. Each voter expresses her preference by ranking the alternatives in a linear order. A *voting rule* takes these preferences as input and selects a single alternative as the winner. The ultimate goal is to choose a socially desirable alternative as the winner.

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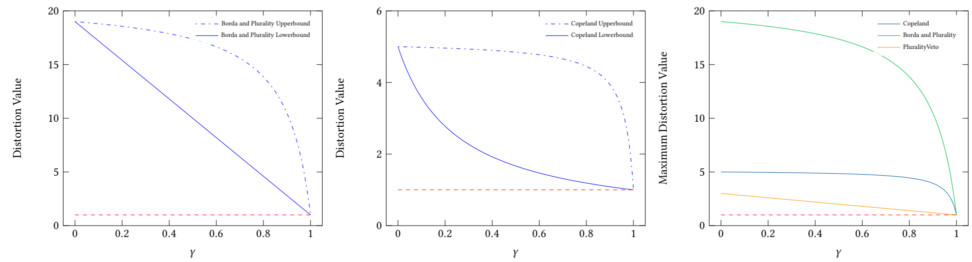
Assessing the efficiency of an alternative in an election requires a systematic evaluation. It is reasonable to consider that every alternative incurs a cost to each voter, and the efficiency of an alternative can be gauged by the total cost they generate for the voters. In this regard, one of the well-established frameworks, namely metric, have been devised to capture these costs [1]. In the metric framework we assume that all voters and alternatives are located within a metric space, and the cost of each alternative for each voter is determined by their respective distances.

Ideally, we would like the voting rule to select the optimal alternative as the winner, that is, the alternative that minimizes the total cost. However, since the voting rule only receives preference lists of the voters as input and lacks knowledge about the underlying costs, it is not always feasible to select the optimal alternative. Technically, this suboptimality in the outcome of the voting rule is due to the information gap of converting ranking-based preferences to cardinal values. The term *distortion* is one of the well-established benchmarks to quantify this gap. For the first time, this term was introduced by Procaccia *et al.* [4]. In the past years, a plethora of studies have been undertaken to quantify the distortion of different voting rules. We refer to a comprehensive survey on distortion by Anshelenich *et al.* [2].

In this paper, our goal is to investigate distortion within the metric framework when voters exhibit "public-spirited" behavior. Public-spirited voting involves voters not only considering their self-interest in alternatives but also the impact of each alternative on the entire population. Recently, Flanigan *et al.* [3] introduced a model to capture public-spirited voters in elections. In this paper, we build upon their model, extending their work to examine how public-spirited voters influence the distortion of different deterministic voting rules within the metric setting.

2 PRELIMINARIES

Every election instance $\mathcal{I} = (\mathcal{V}, \mathcal{A}, \pi, \gamma)$ contains a set $\mathcal{V}$ of n *voters*, a set $\mathcal{A}$ of m *alternatives*, a public-spirited vector $\gamma = [\gamma_1, \ldots, \gamma_n]$, and a preference profile $\pi = (\pi_1, \ldots, \pi_n)$, where π_i is a linear order on the alternatives that shows the preference list of voter v_i. Throughout the paper, we denote the i^{th} voter by v_i, and use symbols like $\mathsf{X}, \mathsf{Y}, \mathsf{Z}$, and W to refer to the alternatives. Following the metric framework, we assume that all the voters and alternatives



(a) Plurality and Borda **(b) Copeland** **(c) Result Comparison**

Figure 1: Distortion across varying levels of public-spiritedness ($0 < \gamma < 1$) for several voting rules. The red dashed line represents the optimal distortion value (i.e. 1) for all values of γ. As γ increases, the distortion of various rules is expected to converge to this line. The gaps between the lower and upper bounds are illustrated for Plurality and Borda in 1a and for Copeland in 1b. Furthermore, the upper bounds across various voting rules are compared in 1c.

are located in some metric space M and denote the distance between voter v_i and alternative X by $\mathsf{d}_{\mathsf{M}}(v_i, \mathsf{X})$, which is equal to their distance.

In this paper, we define a parameter $\gamma_i \in (0, 1)$ for each voter v_i that shows her public-spirited level. For an alternative X, the public-spirited cost of X to voter v_i, denoted by $\textbf{ps-c}_{\mathsf{M}}(v_i, \mathsf{X})$ is defined as $\textbf{ps-c}_{\mathsf{M}}(v_i, \mathsf{X}) = (1 - \gamma_i)\mathsf{d}_{\mathsf{M}}(v_i, \mathsf{X}) + \gamma_i \frac{\textbf{sc}_{\mathsf{M}}(\mathsf{X})}{n}$, where $\textbf{sc}_{\mathsf{M}}(\mathsf{X}) = \sum_{v_i \in \mathcal{V}} \mathsf{d}(v_i, \mathsf{X})$ is the social cost of X. In the public-spirited model, each voter v_i ranks the alternatives according to her public-spirited costs, that is, $\mathsf{X} \prec_{v_i} \mathsf{Y} \iff \textbf{ps-c}_{\mathsf{M}}(v_i, \mathsf{X}) > \textbf{ps-c}_{\mathsf{M}}(v_i, \mathsf{Y})$. We say voter v_i *prefers* alternative X over alternative Y, if $\mathsf{Y} \prec_{v_i} \mathsf{X}$. For an election instance, we define γ_{min} and γ_{max} respectively as the minimum and the maximum value of γ_i among all voters. Also, for any $0 < x < 1$ we define $z(x) = (1 - x)/x$. We frequently use $z(\gamma_{max})$ and $z(\gamma_{min})$ throughout the paper.

Given a voting rule f and an instance $\mathcal{I} = (\mathcal{V}, \mathcal{A}, \pi, \gamma)$, assuming that rule f selects W as the winner of $\mathcal{I}$, we say a metric M is consistent with preference profile π, denoted as $\mathsf{M} \triangleright \pi$, if for every voter v_i and alternatives X and Y, we have $\mathsf{X} \prec_{v_i} \mathsf{Y}$ if and only if $\textbf{ps-c}_{\mathsf{M}}(v_i, \mathsf{X}) > \textbf{ps-c}_{\mathsf{M}}(v_i, \mathsf{Y})$. For an instance $\mathcal{I}(\mathcal{V}, \mathcal{A}, \pi, \gamma)$ and metric $\mathsf{M} \triangleright \pi$, we define the distortion of f as $\mathsf{D}(\mathsf{f}, \mathcal{I}, \mathsf{M}) = \max_{\mathsf{X} \in \mathcal{A}} \frac{\textbf{sc}_{\mathsf{M}}(\mathsf{W})}{\textbf{sc}_{\mathsf{M}}(\mathsf{X})}$. Furthermore, we define $\mathsf{D}(\mathsf{f}, \mathcal{I}) = \max_{\mathsf{M} \triangleright \pi} \mathsf{D}(\mathsf{f}, \mathcal{I}, \mathsf{M})$. Now, let Ω_γ be the set of all possible election instances with public-spirited vector equal to γ. Then, we define the metric distortion of rule f for public-spirited vector γ as $\mathsf{D}_\gamma(\mathsf{f}) = \max_{\mathcal{I} \in \Omega_\gamma} \mathsf{D}(\mathsf{f}, \mathcal{I})$.

3 RESULTS OVERVIEW

In this paper, we explore the impact of public-spirited voting on the distortion in the metric framework. Our study serves as an extension of Flanigan *et al.*'s work [3] into the metric domain. We refer to Table 1 for a summary of our results. First, we provide a general lowerbound of $3 - 2\gamma_{max}$ for the distortion of any deterministic voting rule. Next, we establish upper and lower bounds on the distortion of several voting rules including some positional scoring rules, Copeland and PluralityVeto.

- For the Veto and k-approval rules, our result is the same as [3]: public-spirited behavior does not improve the distortion in the metric setting. For both of these rules, we provide examples which show that the distortion is unbounded.
- For the Plurality and Borda rules, we provide both upper bounds and lower bounds on the distortion value in the metric setting. For Plurality and Borda, we show that the distortion is within range $\left[1 + \frac{2(m-1)}{1+z(\gamma_{max})^{-1}}, 1 + \frac{2(m-1)}{1+(mz(\gamma_{min}))^{-1}}\right]$.
- For the Copeland rule, we prove the upper bound of $1 + \frac{4}{1+(nz(\gamma_{min}))^{-1}}$, and for the case that the public-spirited vector is uniform ($\forall \gamma_i = \gamma_{\text{uni}}$), we provide the lower bound of $\frac{5}{1+4\gamma_{\text{uni}}}$. These results mirror the utilitarian framework: in the metric framework, increased public-spiritedness among voters results in reduced distortion value. For Copeland, we introduce a new technique for proving lower-bound which we call metric-transformation.
- For the PluralityVeto rule, we only provide upper bound on the distortion value. Interestingly, for the case that γ is uniform, this bound matches our general lower bound for any voting rule in the metric framework.

Table 1: Distortion of various voting rules in the metric setting under public-spirited behavior. Some bounds hold for a uniform public-spirited vector ($\forall \gamma_i = \gamma_{\text{uni}}$).

Voting Rule	Metric Distortion	
	lower bound	upper bound
Plurality	$1 + \frac{2(m-1)}{1+z(\gamma_{max})^{-1}}$	$1 + \frac{2(m-1)}{1+(mz(\gamma_{min}))^{-1}}$
Borda	$1 + \frac{2(m-1)}{1+z(\gamma_{max})^{-1}}$	$1 + \frac{2(m-1)}{1+(mz(\gamma_{min}))^{-1}}$
Veto	unbounded	
k-approval	unbounded	
Copeland	$\frac{5}{1+4\gamma_{\text{uni}}}$	$1 + \frac{4}{1+(nz(\gamma_{min}))^{-1}}$
PluralityVeto	$3 - 2\gamma_{max}$	$3 - 2\gamma_{\text{uni}}$

Note that all our upper bounds are parameterized by γ_{min}, while our lower bounds depend on γ_{max}. Consequently, comparing these bounds isn't straightforward. However, in uniform cases, direct comparisons are possible. Figure 1 provides a visual representation of our results.

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