Mechanism Design for Reducing Agent Distances to Prelocated Facilities

Extended Abstract

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ABSTRACT

We consider a variant of facility location problems where the facility is prelocated at a specific position to serve the agents who are located on a real line. Because the facility cannot be relocated due to various constraints (e.g., construction costs and requirements), the social planner considers the structural modification problem of adding short-cut edges to the real line (e.g., shuttles between pairs of locations) for improving the accessibility or reducing costs of the agents to the facility, where the cost of an agent is measured by their shortest distance to the facility possibly using the short-cut edges. We focus on the mechanism design aspects of the problems where the agents' locations are private. We propose several strategy-proof mechanisms that elicit true agent locations and minimize the total or maximum cost of agents. We provide approximation ratios for these mechanisms and lower bounds on the approximation ratios for total or maximum cost.

KEYWORDS

Facility Location, Mechanism Design, Approximation Ratio

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1 INTRODUCTION

Facility location problems (FLPs) have received a lot of attention in recent years within the context of approximate mechanism design without money [1, 8, 15, 17]. In the mechanism design version of the FLPs, a social planner aims to design mechanisms to locate a facility to serve a group of agents located in a metric space (e.g., on a real line) to minimize the cost objective that accounts for the agents' accessibility or distances to the facility. Because each agent's location in the metric space is private, the mechanisms also need to elicit agent location information to determine the optimal location. However, it is known that each agent can misreport their location to bring the facility closer to them by manipulating the mechanisms (e.g., to change their output locations). Therefore, the social planner 's objective is to design a mechanism that is strategy-proof (i.e., the agents have incentives to report their true location information), while approximately minimizing a given cost objective.

Our Problem and Approach. In many situations, a facility (such as a school, park, or library) prelocated in the past can become less than ideal for the agents due to the change in agent compositions, where the population in the targeted domain is different from that in the past (e.g., agents moving into different locations) or change in the social planner's objective. A straightforward approach would be to locate a new facility or relocate the facility to improve the accessibility or distances of the agents to the facility. Unfortunately, the planner is often unable to simply ignore the prelocated facility due to various constraints (e.g., costs of building the facility). In order to improve accessibility for the agents when building a new facility or relocating a facility is not totally feasible, we therefore require an alternative approach.

Existing facility location literature [7, 14] has suggested a *structural modification* approach that alters the metric space to improve the accessibility or reduce the distances of the agents to the facility. For instance, the social planner can construct new edges (such as roads or bridges) or offer shuttle services between two points in the metric space to reduce the distances that the agents must travel to reach the facility [2, 3]. While [7, 14] focused on the optimization aspect of structural modification to reduce agents' distances to the facility, the work of [5] initiated the study of structural modification from the mechanism design without money perspective where agent locations are private. More formally, when the facility has been prelocated, [5] aimed to design strategyproof mechanisms to elicit agent locations on a real line and determine a costless short-cut edge (e.g., a shuttle service) between two points to add to reduce the costs of agents (measured as a total or maximum cost).

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Table 1: A summary of our results for adding one edge.

Objective	Deterministic		Randomized	
	Lower	Upper	Lower	Upper
Max cost	$\frac{2k^2-k+1}{k^2+k}$	$\min\{\frac{3k+1}{k+1}, k\}$	$\frac{3k^2+2k+3}{2k^2+4k+2}$	$\max\{\frac{23}{8}+\frac{5}{8k}, \frac{11}{4}+\frac{1}{k}\}$
Social cost	$\frac{3k^2+k}{2k^2+3k-1}$	$\frac{nk^2+(n-2)k}{k^2+(n-2)k+n-1}$	$\frac{1.02k+1.98}{k+2}$ when $k \geq 2.2$	$\frac{(3n-3)k^3+(n^2+n+2)k^2+(2n^2-3n+1)k+n^2-n}{(n+1)k^3+(n^2+n-2)k^2+(2n^2-n+1)k+n^2-n}$

Table 2: A summary of our results for adding two edges.

Objective	Deterministic		Randomized	
	Lower	Upper	Lower	Upper
Max cost	∞	∞	$\frac{3}{2}$	$3n$
Social cost	∞	∞	1.02	$\frac{(n-2)(4n^2+11n+30)}{n(n+2)(n+6)}$

In this paper, we re-examine the structural modification from the mechanism design without money perspective as in [5]. While the work of [5] focused on the setting of adding a single short-cut edge of incurring zero cost to the agents, we extend it to two natural situations: (i) adding a short-cut edge that has a cost proportional to the (absolute) distance of its two endpoints, and (ii) adding two short-cut edges with zero cost.

2 CONTRIBUTIONS

We consider the problems of designing strategyproof mechanisms to elicit agent private locations and add short-cut edges to reduce agent distances to prelocated facilities on a real line.

Once the short-cut edges are determined, the cost of the agents is the length of their shortest path to the facility, which can use the short-cut edges to reduce the original distance or go to the facility directly in the case that the short-cut edges do not help. The mechanism is required to be *strategy-proof*, which guarantees that no agent can decrease their cost by misreporting the location. Moreover, we are interested in mechanisms that minimize the total cost of all agents (i.e., the *social cost* objective) or minimize the max cost of agents (i.e., the *max cost* objective).

In this paper, we investigate two main directions: adding one short-cut edge with linear cost and adding two short-cut edges with zero cost. Our results are summarized in Tables 1 and 2.

In this setting of adding one short-cut edge with linear cost, there are n agents and a facility prelocated on a real line. Given the reported locations, the mechanism selects two points $a, b \in \mathbb{R}$ and creates a short-cut edge (a, b). The length or the cost that an agent incurs when using the edge (a, b) is $\frac{|a-b|}{k}$, where $k > 1$ is a constant coefficient. Naturally, the (general) cost of the agents is the length of their shortest path to the facility (possibly using the short-cut edge). For social and max cost objectives, we present deterministic and randomized strategy-proof mechanisms with constant approximation ratios. We provide the lower bounds on the approximation ratios. The results are summarized in Table 1.

In the setting of adding two short-cut edges with zero cost, a mechanism selects two edges (a_1, b_1) and (a_2, b_2) with zero cost, that is, the cost that an agent incurs when traveling from a_1 to b_1 (resp. a_2 to b_2) through the short-cut edge is 0. Similarly, the (general) cost of the agents is the length of their shortest path to the facility (possibly using the short-cut edges). For both social and max cost objectives, we present randomized strategy-proof mechanisms and prove the approximation ratios, as well as the lower bounds on the approximation ratios. While it can be difficult to design a deterministic mechanism with a constant approximation ratio, our main contribution in this part is randomized mechanisms. The results are summarized in Table 2.

3 RELATED WORK

We briefly survey facility location studies that are most related to the considered mechanism design setting of structural modification to reduce agents' distances to the prelocated facility.

Procaccia and Tennenholtz [15] initialized approximate mechanism design without money for facility location. This agenda investigates strategy-proof mechanisms through the lens of the approximation ratio. In a typical setting, the agents report their private locations on the real line to a mechanism, and then the mechanism determines the locations for building facilities, where the cost of agents is the distance to the facilities. For the single-facility problem, they provide precise bounds on the approximation ratio of strategy-proof mechanisms for the social cost and maximum cost objectives. Later, [12, 13] improved the bound for problems with two facilities. In addition, a large variety of variations on this typical setting are thoroughly investigated, including different preferences for facilities [10, 16], the distance constraint [6], and other cost functions [9, 11]. We refer readers to a recent survey on more models and results for mechanism design for facility location [4]. While all these studies focus on locating facility locations, our work focuses on how to modify the structure to reduce the costs incurred by agents when the facility has been prelocated.

The work of [5] is the first to study the structural modification for facility location problems from the mechanism design perspective. Precisely, when the facility has been prelocated, [5] aimed to design strategyproof mechanisms to elicit agent locations on a real line and determine a short-cut edge with zero cost between two points. This edge is added to improve the accessibility of agents to the facility and reduce their costs.

REFERENCES

[1] Haris Aziz, Hau Chan, Barton Lee, Bo Li, and Toby Walsh. 2020. Facility location problem with capacity constraints: Algorithmic and mechanism design perspectives. In *Proceedings of the AAAI Conference on Artificial Intelligence*, Vol. 34. 1806–1813.

[2] Oded Berman, Divinagracia I Ingco, and Amedeo Odoni. 1994. Improving the location of minimax facilities through network modification. *Networks* 24, 1 (1994), 31–41.

[3] Oded Berman, Divinagracia I Ingco, and Amedeo R Odoni. 1992. Improving the location of minisum facilities through network modification. *Annals of Operations Research* 40 (1992), 1–16.

[4] Hau Chan, Aris Filos-Ratsikas, Bo Li, Minming Li, and Chenhao Wang. 2021. Mechanism Design for Facility Location Problems: A Survey. In *Proceedings of the 30th International Joint Conference on Artificial Intelligence (IJCAI)*. 4356–4365.

[5] Hau Chan and Chenhao Wang. 2023. Mechanism Design for Improving Accessibility to Public Facilities. In *Proceedings of the 2023 International Conference on Autonomous Agents and Multiagent Systems*. 2116–2124.

[6] Xujin Chen, Xiaodong Hu, Xiaohua Jia, Minming Li, Zhongzheng Tang, and Chenhao Wang. 2018. Mechanism Design for Two-Opposite-Facility Location Games with Penalties on Distance. In *Algorithmic Game Theory*, Xiaotie Deng (Ed.). Springer International Publishing, Cham, 256–260.

[7] Erik D. Demaine and Morteza Zadimoghaddam. 2010. Minimizing the Diameter of a Network Using Shortcut Edges. In *Algorithm Theory - SWAT 2010*, Haim Kaplan (Ed.). Springer Berlin Heidelberg, Berlin, Heidelberg, 420–431.

[8] Elad Dokow, Michal Feldman, Reshef Meir, and Ilan Nehama. 2012. Mechanism design on discrete lines and cycles. In *Proceedings of the 13th ACM Conference on Electronic Commerce (ACM-EC)*. 423–440.

[9] Michal Feldman and Yoav Wilf. 2018. Strategyproof Facility Location and the Least Squares Objective. In *Proceedings of the Fourteenth ACM Conference on Electronic Commerce* (Philadelphia, Pennsylvania, USA) *(EC '13)*. Association for Computing Machinery, New York, NY, USA, 873–890. https://doi.org/10.1145/2482540.2482543

[10] Chi Kit Ken Fong, Minming Li, Pinyan Lu, Taiki Todo, and Makoto Yokoo. 2018. Facility location games with fractional preferences. In *Proceedings of the 32nd AAAI Conference on Artificial Intelligence (AAAI)*. 1039–1046.

[11] Dimitris Fotakis and Christos Tzamos. 2013. Strategyproof Facility Location for Concave Cost Functions. In *Proceedings of the Fourteenth ACM Conference on Electronic Commerce* (Philadelphia, Pennsylvania, USA) *(EC '13)*. Association for Computing Machinery, New York, NY, USA, 435–452. https://doi.org/10.1145/2492002.2482595

[12] Pinyan Lu, Xiaorui Sun, Yajun Wang, and Zeyuan Allen Zhu. 2010. Asymptotically Optimal Strategy-Proof Mechanisms for Two-Facility Games. In *Proceedings of the 11th ACM Conference on Electronic Commerce* (Cambridge, Massachusetts, USA) *(EC '10)*. Association for Computing Machinery, New York, NY, USA, 315–324. https://doi.org/10.1145/1807342.1807393

[13] Pinyan Lu, Yajun Wang, and Yuan Zhou. 2009. Tighter Bounds for Facility Games. In *Internet and Network Economics*, Stefano Leonardi (Ed.). Springer Berlin Heidelberg, Berlin, Heidelberg, 137–148.

[14] Senni Perumal, Prithwish Basu, and Ziyu Guan. 2013. Minimizing eccentricity in composite networks via constrained edge additions. In *IEEE Military Communications Conference (MILCOM)*. IEEE, 1894–1899.

[15] Ariel D. Procaccia and Moshe Tennenholtz. 2013. Approximate Mechanism Design without Money. *ACM Trans. Econ. Comput.* 1, 4, Article 18 (dec 2013), 26 pages.

[16] Paolo Serafino and Carmine Ventre. 2016. Heterogeneous facility location without money. *Theoretical Computer Science* 636 (2016), 27–46.

[17] Taiki Todo, Atsushi Iwasaki, and Makoto Yokoo. 2011. False-name-proof Mechanism Design without Money. In *Proceedings of the 10th International Conference on Autonomous Agents and Multiagent Systems (AAMAS) Volume 2*. 651–658.