Verifying Proportionality in Temporal Voting

Extended Abstract

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ABSTRACT

We study a model of multiwinner voting where candidates are selected sequentially in rounds over a time horizon. Prior work has adapted popular notions of justified representation as well as voting rules that provide strong representation guarantees from the standard single-round multiwinner election case to the temporal setting. In our work, we focus on the complexity of verifying whether a given outcome is proportional. We show that the temporal setting is strictly harder than the standard single-round model of multiwinner voting, but identify natural special cases that enable efficient verification.

KEYWORDS

Temporal Voting, Proportionality, Computational Social Choice

ACM Reference Format:

Edith Elkind, Svetlana Obraztsova, and Nicholas Teh. 2024. Verifying Proportionality in Temporal Voting: Extended Abstract. In *Proc. of the 23rd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2024), Auckland, New Zealand, May 6 – 10, 2024*, IFAAMAS, 3 pages.

1 INTRODUCTION

Consider a large corporation that has decided to improve its public image and to give back to the society by engaging in *corporate philanthropy* over the next decade, committing a small fraction of their profits towards supporting the efforts of a single charitable organization, to be selected on an annual basis. The management of this corporation decides to ask its customers, staff, and shareholders for input as to which charity organizations it should select each year. Furthermore, as the charity selected is of strategic importance and would directly impact the company's corporate image and hence profitability, it is important for the company to ensure that the selection is representative of what its customers, staff, and shareholders care about and that it would create maximum impact for the charitable organization they choose to support.

It is natural to view this problem through the lens of multiwinner voting [15, 19, 26], where several notions of representation and fairness have been proposed over the past decade, spanning across proportional representation [4, 15], diversity [9, 31], and excellence, amongst others [26]. Perhaps the most prominent among these is the concept of *justified representation (JR)* and its variants (such as proportional justified representation (PJR) and extended justified representation (EJR)), which aim to capture the idea that large cohesive groups of voters should be fairly represented in the final outcome [2, 3, 10, 29, 32] . However, these notions of fairness do not fully capture settings where preferences may evolve over time.

Temporal considerations in multiwinner voting have been studied recently, most notably in the line of work known as *perpetual voting* [22, 23], and have broad real-world applications [17]. Thus, it would be appropriate to consider models whereby preferences are elicited over a specific time period, and could change. Accordingly, suitable notions of fairness should be defined to take into account temporal considerations. In this spirit, Bulteau et al. [11] began a foray into defining proportional representation in the setting of temporal multiwinner voting with approval preferences. They proposed multiple temporal variants of JR and PJR, and studied their existence and computational complexity.

Concurrently and independently from our work, Chandak et al. [12] delve into the study of notions proposed in Bulteau et al. [11], and study adaptations of existing rules from single-shot elections to the temporal case, for both online and offline settings. Bredereck et al. [7, 8] look at sequential committee elections whereby an entire committee is elected in each round, and impose constraints on the extent a committee can change, whilst ensuring candidates retain sufficient support from the electorate. Lackner et al. [25] propose a framework for studying long-term participatory budgeting, and study fairness considerations in that setting. Other models in the social choice literature that include temporal elements include public decision-making [6, 13, 18, 24, 33], scheduling [16, 28], resource allocation over time [1, 5, 21], online committee selection [14], and dynamic social choice [20, 27].

2 PRELIMINARIES

A *temporal election* is a tuple $(N, P, \ell, (\mathbf{s}_i)_{i\in N})$, where $N = \{1, \ldots, n\}$ is a set of *voters*, $P = \{p_1, \ldots, p_m\}$ is a set of m distinct *projects*, or *candidates*, ℓ is the number of *timesteps*, and, for each $i \in N$, $\mathbf{s}_i = (s_{i,1}, s_{i,2}, \ldots, s_{i,\ell})$, where $s_{i,t} \subseteq P$ is the *approval set* of voter i at time t, which consists of candidates that i approves at timestep t. We refer to $\mathbf{s}_i$ as i's *temporal preference*; for brevity, we will sometimes omit the term "temporal". An *outcome* of a temporal election $(N, P, \ell, (\mathbf{s}_i)_{i\in N})$ is a sequence $\mathbf{o} = (o_1, \ldots, o_\ell)$ of ℓ candidates such that for every $t \in [\ell]$ candidate $o_t \in P$ is chosen at timestep t. A candidate may be selected multiple times. A voter i's *utility* for an outcome $\mathbf{o}$ is computed as $u_i(\mathbf{o}) = |\{t \in [\ell] : o_t \in s_{i,t}\}|$.

Definition 2.1. Given an election $E = (N, P, \ell, (\mathbf{s}_i)_{i\in N})$ and a group of voters $N' \subseteq N$, we define the *agreement* of N' as the number of timesteps in which all members of N' approve a common candidate: $\beta(N') = |\{t \in [\ell] : \cap_{i\in N'} s_{i,t} \neq \varnothing\}|$. We define the *demand* of a group of voters N' as $\alpha(N') = \left\lfloor \beta(N') \cdot \frac{|N'|}{n} \right\rfloor$.

Proc. of the 23rd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2024), N. Alechina, V. Dignum, M. Dastani, J.S. Sichman (eds.), May 6 – 10, 2024, Auckland, New Zealand. © 2024 International Foundation for Autonomous Agents and Multiagent Systems (www.ifaamas.org).

Definition 2.2 (Justified Representation). An outcome $\mathbf{o}$ satisfies *justified representation (JR)* for an election $E = (N, P, \ell, (\mathbf{s}_i)_{i\in N})$ if for every group of voters $N' \subseteq N$ with $\alpha(N') > 0$ we have $|\{t \in [\ell] : o_t \in \bigcup_{i\in N'} s_{i,t}\}| > 0$.

Definition 2.3 (Proportional Justified Representation). An outcome $\mathbf{o}$ satisfies *proportional justified representation* (PJR) for an election $E = (N, P, \ell, (\mathbf{s}_i)_{i\in N})$ if for every group of voters $N' \subseteq N$ we have $|\{t \in [\ell] : o_t \in \bigcup_{i\in N'} s_{i,t}\}| \geq \alpha(N')$.

Definition 2.4 (Extended Justified Representation). An outcome $\mathbf{o}$ satisfies *extended justified representation* (EJR) for an election $E = (N, P, \ell, (\mathbf{s}_i)_{i\in N})$ if for every group of voters $N' \subseteq N$ there exists a voter $i \in N'$ such that $|\{t \in [\ell] : o_t \in s_{i,t}\}| \geq \alpha(N')$.

It is easy to see that EJR implies PJR, and PJR implies JR[1].

3 RESULTS

We start by showing that a variant of Greedy Cohesive rule [29] always produces an outcome that satisfies EJR (and hence PJR and JR); briefly, this rule considers all subsets of voters in a certain order, picks a collection of pairwise disjoint subsets, and then allocates a timestep to each of these subsets.

Theorem 3.1. *There exists an algorithm that returns an outcome satisfying EJR, and runs in time $O(2^n \cdot poly(n, m, \ell))$.*

Given that EJR outcomes are guaranteed to exist, it is natural to ask whether we can efficiently compute an EJR outcome or verify that a given outcome satisfies EJR (or one of the weaker axioms). The former question is addressed by Chandak et al. [12], so we mostly focus on the latter question.

In single-shot multiwinner elections, outcomes that satisfy EJR (and thus JR and PJR) can be computed in polynomial time [3, 30]. In contrast, the problem of verifying if a given outcome satisfies JR/PJR/EJR is considerably more challenging: while this problem is polynomial-time solvable for JR, it is coNP-hard for PJR and EJR [3, 32]. We show that in the temporal setting, the verification problem is coNP-hard for JR, PJR, and EJR.

Theorem 3.2. *For each of $X \in$ {JR, PJR, EJR}, verifying whether an outcome satisfies X is* coNP*-complete, even if $|P| = 2$.*

Our proof establishes that verifying JR remains coNP-hard even if there are just two distinct projects. However, the election constructed in our reduction has the property that the approval sets $s_{i,t}$ may be empty. If we require that $s_{i,t} \neq \emptyset$ for all $i \in N$, $t \in [\ell]$ then for $|P| = 2$ the problem of verifying JR becomes easy.

Proposition 3.3. *Given an election $(N, P, \ell, (\mathbf{s}_i)_{i\in N})$ with $P = \{p, q\}$ and $s_{i,t} \neq \emptyset$ for all $i \in N$, $t \in [\ell]$, as well as an outcome $\mathbf{o}$, we can check in polynomial time whether $\mathbf{o}$ provides JR.*

This shows that requiring each voter to approve at least one project at each step may reduce the complexity of checking JR considerably, at least for $|P| = 2$. Is this still the case for $|P| > 2$? If there are no constraints on the size of P, the answer is 'no'.

Proposition 3.4. *For each of $X \in$ {JR, PJR, EJR}, verifying whether an outcome satisfies X is* coNP*-complete, even if $s_{i,t} \neq \emptyset$ for all voters $i \in N$ and all timesteps $t \in [\ell]$.*

[1] Our terminology differs from Chandak et al. [12], who refer to the notions defined above as "strong JP/PJR/EJR", and reserve the terms JR/PJR/EJR for weaker concepts.

It remains an open question whether verifying that a given outcome provides JR remains coNP-complete if all approval sets are non-empty and $|P|$ is a fixed constant greater than 2.

We also note that we can obtain a hardness-of-verification result for $|P| = 3$ under the assumption that all approval sets are non-empty for a weaker notion of JR, which was also proposed as *all-period intersection JR* by Bulteau et al. [11]. Formally, an outcome $\mathbf{o}$ provides *all-periods Justified Representation* for an election $E = (N, P, \ell, (\mathbf{s}_i)_{i\in N})$ if for every group of voters $N' \subseteq N$ with $\beta(N') = \ell$, $\alpha(N') > 0$ we have $|\{t \in [\ell] : o_t \in \bigcup_{i\in V} s_{i,t}\}| > 0$. We note that Chandak et al. [12] also consider this concept as refer to it as JR (and use the term 'strong JR' for what we call JR).

Theorem 3.5. *Verifying whether an outcome satisfies all-periods JR is coNP-complete, even if $|P| = 3$ and $s_{i,t} \neq \emptyset$ for all voters $i \in N$ and all timesteps $t \in [\ell]$.*

We complement our hardness results with the following fixed-parameter tractability results.

Proposition 3.6. *For each of $X \in$ {JR, PJR, EJR}, verifying whether an outcome satisfies X is FPT with respect to n, FPT with respect to the combined parameter (m, ℓ), and XP with respect to ℓ.*

As the problem of verifying whether an outcome satisfies JR, PJR, or EJR is computationally hard in general, it is natural to seek a restriction on voters' preferences that may yield positive results. We consider the case where voters' preferences evolve monotonically with time: that is, for any two timesteps $t, t' \in [\ell]$ with $t < t'$ and every project $p \in P$ it holds that if $p \in s_{i,t}$ for some agent $i \in N$, then $p \in s_{i,t'}$. We call this the *structured availability* setting.

Theorem 3.7. *In the structured availability setting, for each of $X \in$ {JR, PJR, EJR}, the problem of verifying whether an outcome satisfies X admits a polynomial-time algorithm.*

We also show an algorithm for finding EJR outcomes that is based on integer linear programming (ILP). While this algorithm does not run in polynomial time, it is very flexible: e.g., we can easily modify it so as to find an EJR outcome that maximizes the utilitarian social welfare, or provides utility guarantees to individual voters.

Theorem 3.8. *There exists an integer linear program (ILP) whose solutions correspond to outcomes that satisfy EJR; the number of variables and the number of constraints of this ILP are bounded by a function of the number of voters n.*

The following corollary illustrates the power of the ILP-based approach. Note that while Chandak et al. [12] show that a variant of the PAV rule can find EJR outcomes in polynomial time, their approach cannot handle additional constraints on agent' utilities and hence the corollary is not implied by their result.

Corollary 3.9. *Consider a temporal election $E = (N, P, \ell, (\mathbf{s}_i)_{i\in N})$. There is an algorithm that is FPT with respect to the number of voters n that for each set of integers $\delta_1, \ldots, \delta_n$ decides whether there exists an EJR outcome of E that guarantees utility δ_i to voter i for each $i \in N$, and, if yes, finds an outcome that maximizes the utilitarian social welfare among all outcomes with this property.*

ACKNOWLEDGMENTS

This work was supported by the AI Programme of The Alan Turing Institute.

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